STRUCTURAL DESIGN

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1. Basic Concepts

1.1 Introduction:

Structural Engineering

Analysis (SOM, SA)	Design (RCC, PSC, Steel)
- Equilibrium	- Safety
- Compatibility	- Serviceability
- Energy	- Durability
	- Economy
	- Asthetic

i> Safety:

A structure must be safe with appropriate factor of safety [FOS] for loading that may come on it during its intended life.

ii> Serviceability:

A structure should provide the service for which it is constructed.

iii> Durability:

A structure should sustain loading for which it was designed and should perform well with safety and serviceability upto its whole life

Durability without serviceability or less margin of safety [FOS] iv>has no meaning

iv> Economy:

Design and construction of any structure should be economical without affecting safety, serviceability and durability.

v> Asthetic:

If huge investment is involved in design and construction of a structure then asthetic also plays an important role.

Ex. Considering a beam:

i) Safety: Reinforcement is provided.

ii) Serviceability: Doubly reinforced section instead of singly reinforced section to reduce depth of section.

iii) Durability: Nominal cover, selection of material.

iv) Economy: Monolythic casting of beam and slab designed as T-section.

v) Asthetic: Half round section instead of rectangular section.

1.2 Cement Concrete:

It is a mixture of binding material [cement], fine aggregate [sand], coarse aggregate, water and admixture in proper proportion to achieve concrete of desired properties at fresh state and hardened state.

1.2.1 Concrete Mix:

a) Nominal Mix:

- Based on experience.
- Mixing may be by weight or by volume. By weight is preferable
- Quantity of water is not fixed. It is provided as per site requirement.
- Nominal mix is allowed for M5 to M20.

	C	FA	CA
M10	1	3	6
M15	1	2	4
M20	1	1.5	3

b) Design Mix:

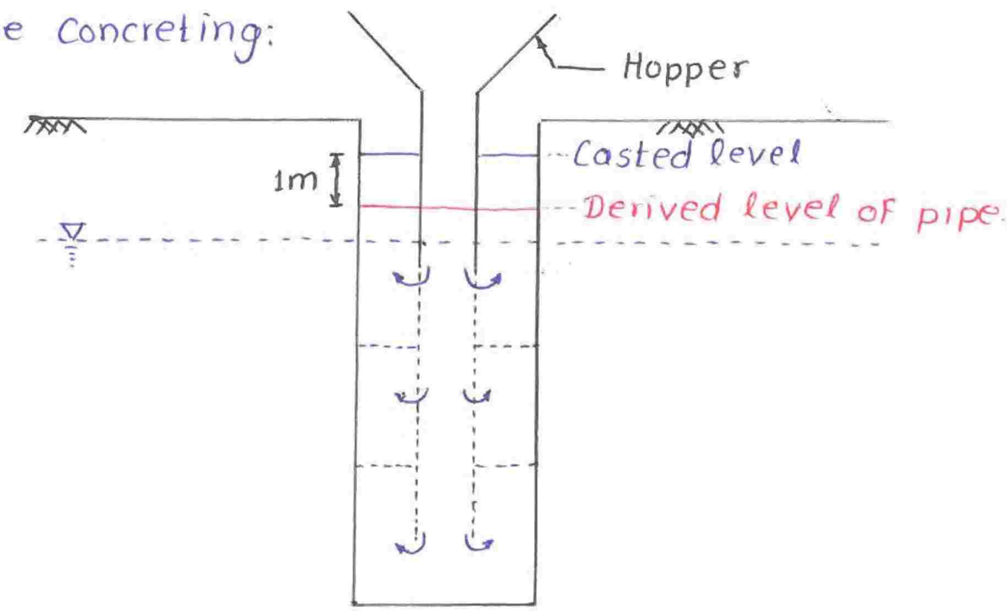
- Based on calculation as per IS10262 (2009)
- Proportioning must be by weight.
- Quantity of water is also fixed.
- Design mix is allowed for M10 to M100.

1.2.2 Fresh Concrete:

Workability is the most important property of fresh concrete which is simply defined as "Ease to work with."

Sr. No.	Degree of Workability	Use	Slump	Compacting Factor	Vee-bee time (sec)
1.	Very low	- Road Construction. - Shallow Section.	-	0.75 - 0.8	10 - 20
2.	Low	- Mass concreting. - Lightly reinforced section	25 - 75	0.8 - 0.85	5 - 10
3.	Medium	- Heavily reinforced section - Concreting by concrete pump.	50 - 100	0.85 - 0.92	2 - 5
4.	High	- Piling	100-150	0.92-above	-
5.	Very High	- Tremie pipe concreting.	-	0.92-above	-

* Tremie Pipe Concreting:



* Workability of Concrete can be measured by following methods.

1. Slump test
2. Compacting factor Test
3. Vee-bee Test
4. Flow Test

1.2.3 Hardened Concrete:

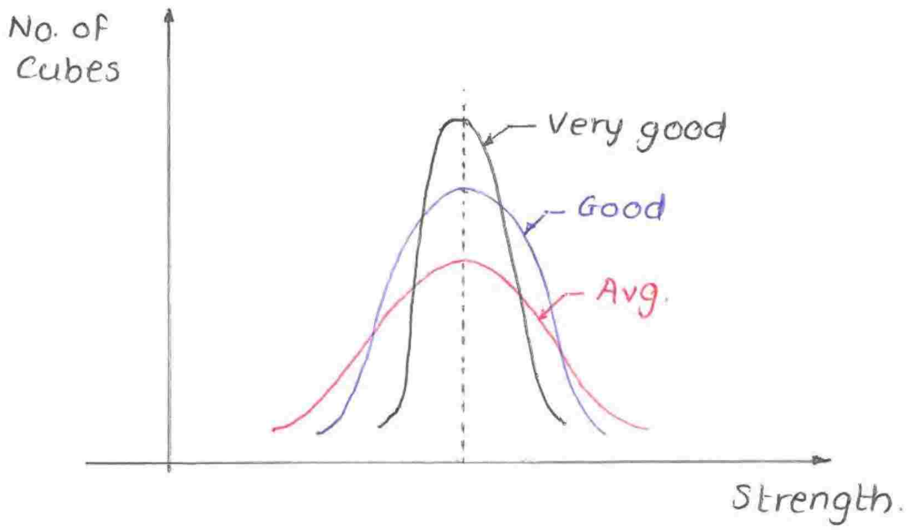
After final setting time, concrete is assumed to be hard and it keeps on gaining strength for very long time [1 to 5 years]

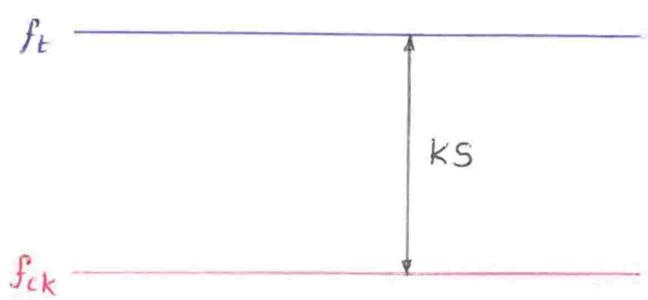
a) Compressive Strength of Cube:

This is the compressive strength of cube size 150 mm subjected to uniaxial compression after 28 days from day of casting.

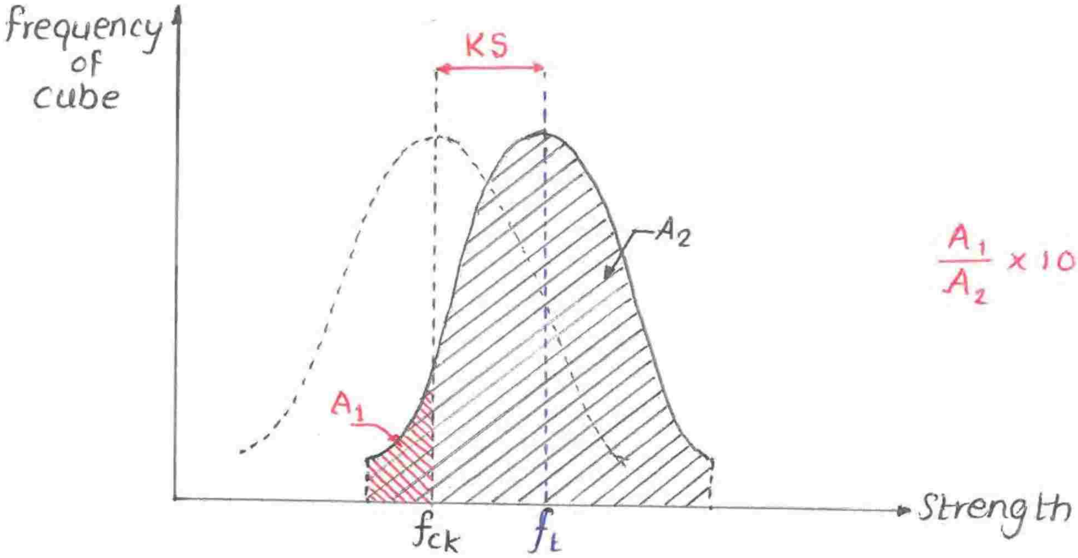
b) Characteristic Compressive Strength of Cube:

It is the strength below which not more than 5% test results are expected to fall.





$$f_t = f_{ck} + KS$$



$\frac{A_1}{A_2} \times 100 \leq 5\%$

Area under curve represents number of cubes.

K = 1.65 (for 5% of definition)

% of definition	K
0%	∞
5%	1.65
50%	0

S = Standard deviation that depends on quality control.

Ex. Uniaxial compression test results of 100 cubes are listed below in increasing order. Find f_{ck}

26, 26.5, 26.5, 27, 27.5,

28, 28.5, 29, 30, 30.5,

31, ...

.................., 42.5 N/mm²

⇒ As per definition, f_{ck} should be 28 N/mm². Since, f_{ck} always designated in multiple of 5, so answer should be 25 N/mm² or 30 N/mm².

In this case, 8 samples (more than 5%) are below 30 N/mm², so 30 N/mm² can not be f_{ck}

Now, 25 N/mm² can be considered as f_{ck} because zero test results (less than 5%) is below 25 N/mm²

$\Rightarrow f_{ck} = 25\ N/mm^2$

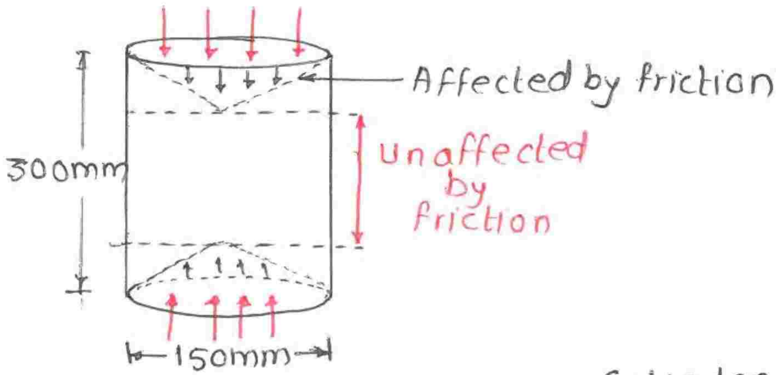
c) Characteristic Compressive Strength of Concrete:

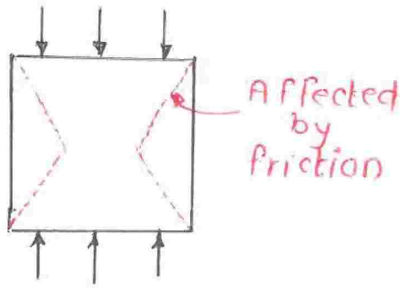
It is obtained by dividing characteristic compressive strength of cube by a factor 1.5 to account for variation in shape of concrete [other than cube] and variation in loading condition [other than uniaxial compression].

* Note:

- Factor 1.5 used here is not partial F.O.S.
- For general conversation, characteristic strength of concrete represents value obtained from characteristic strength of Cube.

1.2.4 Comparison between Cube and Cylinder:





	Cylinder	Cube
What should be used	✓	×
Actually Used.	×	✓

- Uniaxial compressive strength of concrete can be determined by using different shapes of spacimen. (Cube, cylinder, prism, etc)

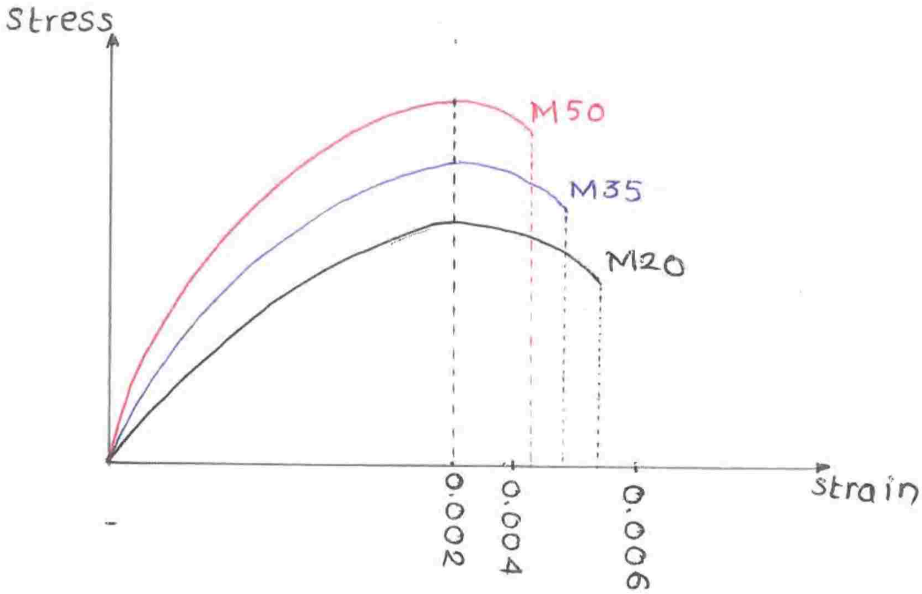
$$f_{cube} \approx 1.25\, f_{cylinder}$$

- Cylinder gives more appropriate results for uniaxial compressive strength of concrete because effect of friction between machine plates and specimen, is almost nil (zero).

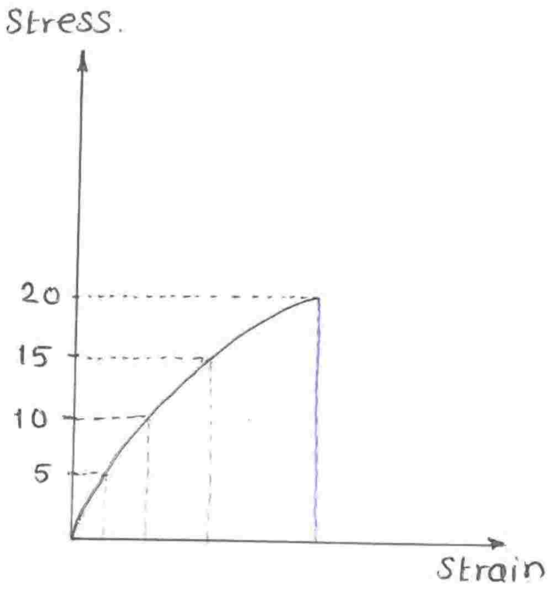
* Note:-

- Cube of smaller size (assuming 100mm) gives more strength than standard cube.
- A smaller cylinder also gives higher strength than standard cylinder, provided ratio of height to diameter remains constant
- These results are experimental.

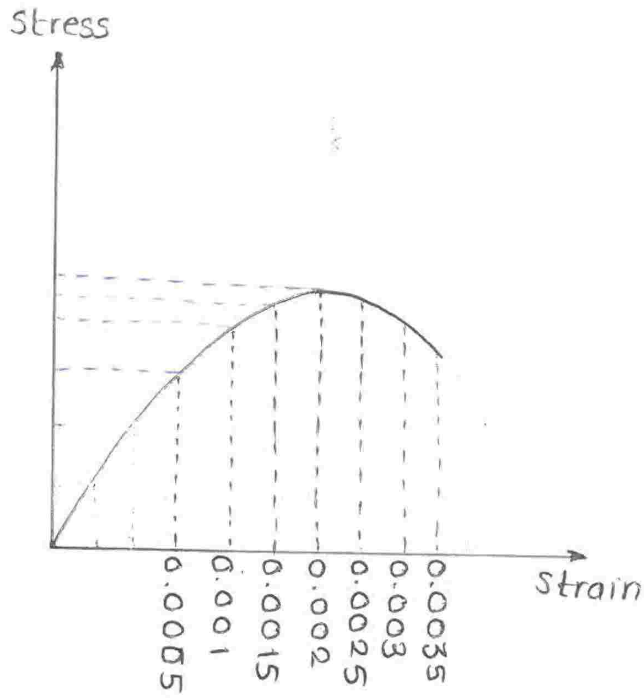
1.2.5 Stress-Strain diagram of Concrete under



- Stress-strain diagram is non-linear.
- Initial portion of stress-strain diagram can be considered as linear.
- Maximum compressive stress is corresponding to approx strain 0.002
- Ultimate strain lies between 0.004 to 0.006
- Modulus of elasticity increases with increase in grade of concrete.
- Brittleness increases with increase in grade of concrete.



Controlled Stress.



Controlled Strain

1.2.6 Grade of Concrete:

$\underbrace{M}_{\text{Mix}} - \underbrace{25}_{\text{Characteristic compressive strength } (N/mm^2)}$

M5 - M20 ⟶ Nominal Mix

M10 - M100 ⟶ Design Mix [as per ammendment (4)]

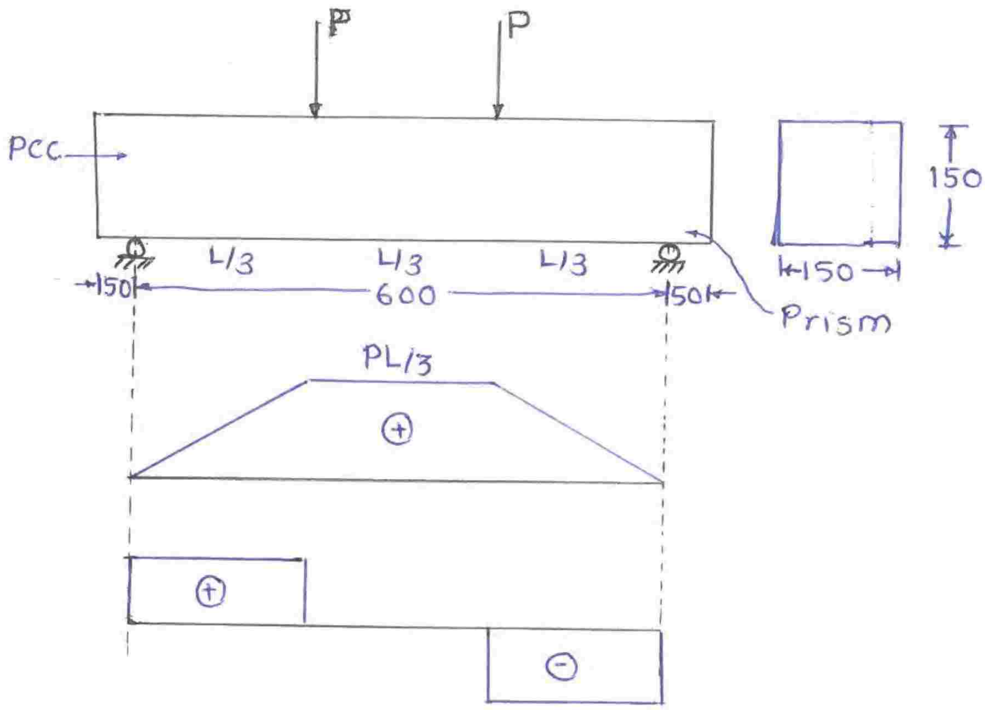
1.2.7 Tensile Strength of Concrete:

- It is approximately 10% (7% to 15%) of the compressive strength.
- Stress-strain diagram is almost linear.
- Ratio of compressive strength to tensile strength increases with increase in grade of concrete.
- Since tensile strength of concrete is ignored in RCC structure so it has very less importance. However, it is calculated to determine cracking moment.

1. Direct Tension Test:

Practically, it is very difficult to perform direct tension test because force never remains perfectly axial tension due to non-homogenity of concrete.

2. Flexure Test



Flexure Formula:

$$\underbrace{\frac{M}{I} = \frac{f}{y}}_{} = \frac{E}{R}$$

$$\frac{P_{cr} \cdot L/3}{bD^3/12} = \frac{f_{cr}}{D/2}$$

$$f_{cr} = ??$$

- 3rd point loading is applied for pure bending condition. (Flexure).
- Value of P is increased from 0 to value corresponding to which 1st crack develops in extreme tension fibre.
- Corresponding to cracking load, bending moment is calculated in central portion and tensile strength is calculated as illustrated above.

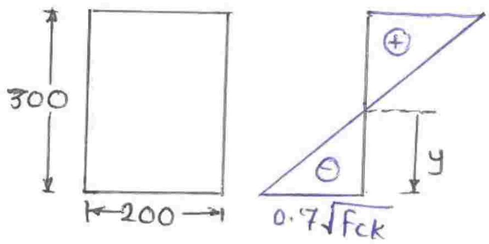
- IS 456 provides standard formula for flexure tensile Strength/Modulus of Rupture

$$f_{cr} = 0.7\sqrt{f_{ck}} \quad N/mm^2.$$

Ex. A PCC beam of section size 200 x 300 mm is made up of M30 concrete. Calculate cracking moment of section.

⇒

By Flexure Formula.



$$\frac{M_{cr}}{I} = \frac{f_{cr}}{y}$$

$$f_{cr} = 0.7\sqrt{f_{ck}}$$

$$f_{cr} = 0.7\sqrt{30} = 3.834 \, N/mm^2$$

$$I = \frac{bD^3}{12} = \frac{200 \times 300^3}{12}$$

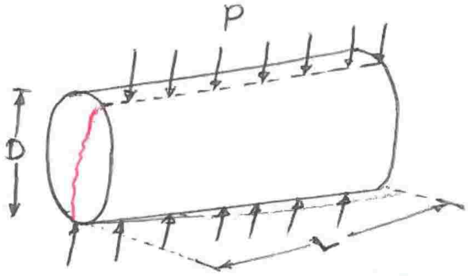
$$I = 450 \times 10^6 \, mm^4$$

$$y = D/2 = 150 \, mm$$

$$\frac{M_{cr}}{450 \times 10^6} = \frac{3.834}{150}$$

$$M_{cr} = 11.502 \, kN\text{-}m$$

3. Cylinder Split Test:
   - A line loading along length is applied at diametrically end points.
   - Due to this loading, Cylinder splits into two parts.



$$f_{cr} = \frac{2P}{\pi D L}$$

$$f_{flexure} > f_{cylinder\ split} > f_{direct}$$

1.2.8 Shrinkage of Concrete:

Concrete has shrinkage property due to presence of cement. IS 456 provides a standard value of shrinkage strain of concrete for design of RCC structure.

$$\epsilon_{st} = 0.0003$$
$$= 0.03\%$$
$$= 3 \times 10^{-4}$$

1.2.9 Durability:

Exposure Condition	Description	Minimum Grade of Concrete	Minimum Nominal Cover	Minimum Cement content	Maximum W/C ratio
Mild	- Protected from rainfall.	M20	20mm*	300 kg/m^3	0.55
Moderate	- Subjected to normal rainfall - Permanently submerged in normal water - Foundation in non aggressive soil.	M25	30mm	300 kg/m^3	0.5
Severe	- Coastal area - Subjected to Heavy rainfall - Permanently submerged in sea water - Alternate drying and wetting in normal water - Occasional freezing.	M30	45mm**	320 kg/m^3	0.45
Very Severe	- Subjected to sea spray (Alternate drying and wetting in sea water) - Permanent freezing.	M35	50mm	340 kg/m^3	0.45

Exposure Condition	Description	Minimum Grade of Concrete	Minimum Nominal Cover	Minimum Cement Content	Maximum W/C ratio
Extreme	- Tidal zone - Subjected to aggressive chemicals	M40	75mm	$360 kg/m^3$	0.4

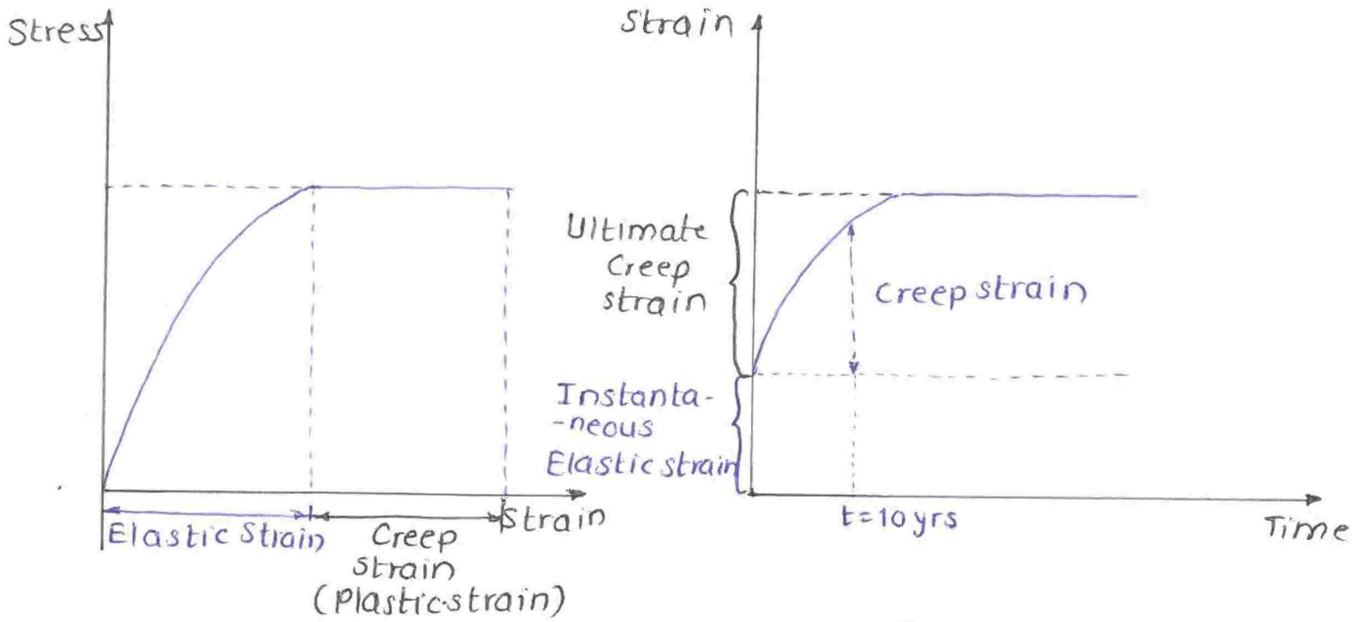
* - This can be reduced to 15mm for reinforcing bar dia. 12mm or less.

** - This can be reduced to 40mm for grade of concrete M35 or higher.

1.2.10 Creep of Concrete:

It is a time dependent strain in concrete due to sustained loading (permanent loading). Dead load and prestressing force are the examples of permanent loading.

The exact mechanism of creep in concrete is still not fully understood. It is generally attributed to internal movement of absorbed water, viscous flow or sliding between the gel particles, moisture loss and the growth in micro-cracks.

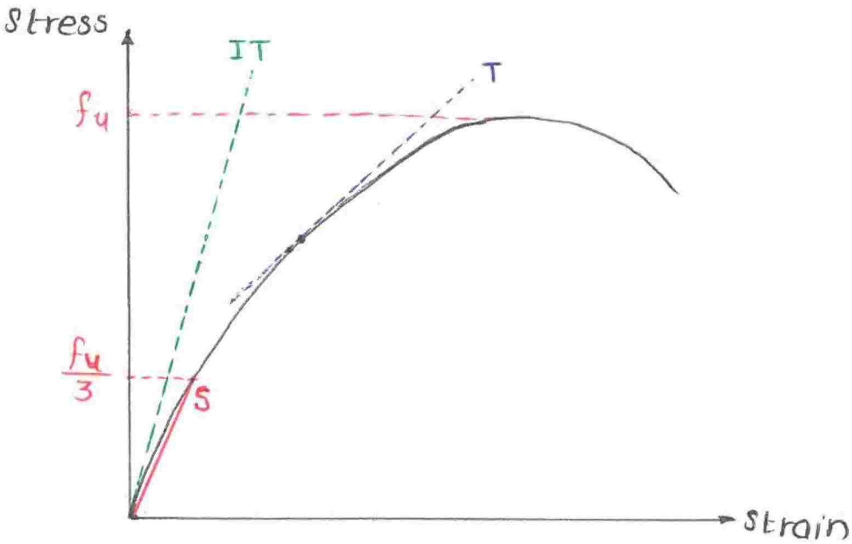


$$\text{Creep coefficient } (\theta) = \frac{\text{Ultimate Creep Strain}}{\text{Instantaneous Elastic Strain}}$$

Age of concrete at the time of loading	θ
7 days	2.2
28 days	1.6
1 Year	1.1

* Note:
  - Higher value of θ represents higher creep strain
  - Ultimate creep strain is more if load is applied at early age of concrete.

1.2.11 Modulus of Elasticity:



1. Initial Tangent Modulus (IT):

It is defined as slope of tangent drawn at a point of start of stress-strain diagram. This is also called as dynamic modulus of elasticity which is used for analysis due to dynamic loading. It is calculated using Resonance frequency test.

2. Secant Modulus (S):

It is defined as slope of line connecting point of start of stress-strain diagram to point with stress $\frac{1}{3}$rd of ultimate stress. It is also called as static modulus of elasticity. IS 456 provides standard formula for this modulus of elasticity.

$$E_c = 5000\sqrt{f_{ck}} \ \text{N/mm}^2$$

This is also called as short term modulus of elasticity and it is not incorporating effect of creep.

After effect of creep.

$$E_{ce} = \frac{E_c}{1+\theta} = \frac{5000\sqrt{f_{ck}}}{1+\theta}$$

3. Tangent Modulus (T):

It is defined as slope of tangent drawn at any point of stress-strain diagram. It is used in incremental load analysis.

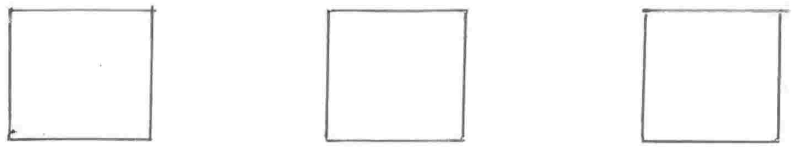
1.2.12 Acceptance of Concrete:

1.2.12.1 Acceptance Criteria of Sample:

- Specimen ⟶ 1 Cube
- Sample ⟶ Set of 3 cubes casted from same concrete at same time.

Quantity of Concrete	No. of Sample
1 - 5 m^3	1
6 - 15 m^3	2
16 - 30 m^3	3
31 - 50 m^3	4
51 - above	4 + 1 sample for each additional 50 m^3.

1 Sample



Compressive Strength of specimen	x	y	z

$$\text{Compressive strength of Sample} = \frac{x+y+z}{3}$$

Sample is acceptable only if compressive strength of no individual specimen (x, y, z) is falling beyond $\pm$ 15% of compressive strength of sample $\left(\frac{x+y+z}{3}\right)$

1.2.12.2 Acceptance Criteria of Concrete

$V = 100\,m^3$

Day	Day 1	Day 2	Day 3	Day 4
Quantity	$20\,m^3$	$35\,m^3$	$25\,m^3$	$20\,m^3$
No. of Sample	3	4	3	3

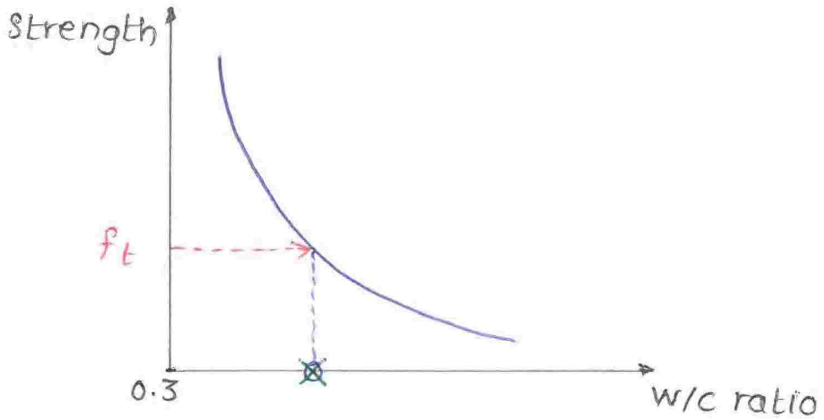
Sample Number. 1 2 3 4 5 6 7 8 9 10 11 12 13

Avg. of 4 consecutive and non-overlapping $\geq$ Maximum $\begin{cases} \bullet\ f_{ck} + (0.825 \times \text{Standard deviation}) \\ \bullet\ f_{ck} + 3 \end{cases}$

In addition to above criteria, compressive strength of no individual sample (1 to 13) should be less than $f_{ck} - 3$

Grade of concrete	S
M10 - M15	3.5
M20 - M25	4.0
M30 - M50	5.0

Step2: Take value of water cement ratio corresponding to target mean strength from graph given in SP23 [Special Publication]



W/c ratio should not be more than value obtained from above graph and corresponding to exposure condition.

Step3: Take maximum water content from Table 2 of IS 10262, corresponding to nominal size of coarse aggregate and slump 25-50 mm

Nominal Size of Coarse Aggregate	Maximum Water Content
10 mm	205 kg/m^3
20 mm	186 kg/m^3
40 mm	165 kg/m^3

*Note:

- Maximum limit of water content is imposed to prepare economical and durable concrete with low shrinkage.
- As nominal size of C.A. increases, water requirement decreases because total surface area of larger size coarse aggregate is less than as of smaller size C.A.
- For each additional 25mm slump, above values are increased by 3%.
- Above values are reduced by 5-10% for plasticizers and 20% for super-plasticizers.
- Maximum limit of plasticizers and Super-plasticizers are 1% and 2% of quantity of cement respectively.

Step 4: Calculate Cement content:

$$\text{Weight of Cement} = \frac{W_w}{W/C}$$

This value should not be less than minimum cement content corresponding to exposure condition.

Step 5: Take proportion of volume of C.A. in volume of total aggregate (F.A.+C.A.) from Table 3 of IS 10262 for W/C ratio 0.5

$$\text{Proportion} = \frac{\text{Volume of Coarse Aggregate}}{\text{Volume of Total Aggregate}} \quad (\text{for } W/C = 0.5)$$

Nominal Size of C.A.	Zone IV	Zone III	Zone II	(Finest) Zone I
10 mm	0.50	0.48	0.46	0.44
20 mm	0.60	0.64	0.62	0.60
40 mm	0.75	0.73	0.71	0.69

For w/c ratio other than 0.5, above values are modified as given below –

± 0.01 in proportion for each ∓ 0.05 in w/c ratio.

*Step 6: Calculate quantity of F.A. and C.A.

Total volume of concrete = $1 m^3$.

(a) Volume of concrete mass = 1 − air content

(b) Volume of Cement = $\frac{W_c}{\text{density}} = \frac{W_c}{S_c \times \rho_w} = \frac{W_c}{S_c \times 1000}$

(c) Volume of Water = $\frac{W_w}{1000}$

(d) Volume of Admixture = $\frac{W_{adm}}{S_{adm} \times 1000}$

(e) Volume of Total aggregate = (a) − (b + c + d)

(f) Weight of Coarse Aggregate = Volume of C.A. × Density

= proportion × (e) × S_{CA} × 1000

* proportion = $\frac{\text{Volume of C.A}}{\text{Volume of T.A.}}$

⇒ Volume of C.A. = proportion × vol. of T.A.

(g) Weight of Fine Aggregate = Volume of F.A × Density

= [1 − proportion] × (e) × S_{FA} × 1000.

Ex. M40, CA = 20 mm, Severe exposure, slump 100 mm, 1% plasticizer, $S_c = 3.15$, $S_{CA/FA} = 2.74$, $S_{adm} = 1.145$, FA of zone I, Air content 1.5%

⇒

Step 1: Target Mean Strength:

$$f_t = f_{ck} + K \cdot S$$

$$= 40 + 1.65 \times 5$$

$$\Rightarrow \boxed{f_t = 48.25 \text{ N/mm}^2}$$

Step 2: W/C ratio:

$W/C = 0.4$ .... [from SP23, $f_t = 48.25$ N/mm²]

This is less than 0.45 (severe) so it is OK.

$$\Rightarrow \boxed{W/C = 0.4}$$

Step 3: Weight of Water:

$W_w = 186$ kg/m3 ..... [from Table 2 of IS 10262, CA = 20 mm]

For 100 mm slump,

$$W_w = 186 + 186 \times \frac{6}{100}$$

$$W_w = 197.16 \text{ kg/m}^3$$

For use of plasticizer

$$W_w = 197.16 - 197.16 \times \frac{8}{100}$$

$$W_w = 181.37 \text{ kg/m}^3$$

This value is maximum limit, so taking

$\Rightarrow \boxed{W_w = 170 \text{ kg/m}^3}$ .... [Based on experience]

Step 4: Cement Content:

$$W_c = \frac{W_w}{W/C} = \frac{170}{0.4} = 425 \text{ kg/m}^3.$$

This is greater than 320 kg/m3 (Severe) so, it is OK.

$$\Rightarrow \boxed{W_c = 425 \text{ kg/m}^3.}$$

Step 5: Proportion:

Proportion = 0.6 ....[from Table 3 of IS 10262 CA. = 20mm & zone I]

For w/c = 0.4,

$$\text{proportion} = 0.6 + (2 \times 0.01) = 0.62$$

$\Rightarrow$ $\boxed{\text{proportion} = 0.62}$

w/c	proportion
0.50	0.60
− 0.05	+ 0.01
− 0.05	+ 0.01
0.40 →	0.62

Step 6: Quantity of F.A. and C.A.

Total Volume of Concrete = 1 m^3

(a) Volume of concrete mass = 1 − air content

$$= 1 - 1 \times \frac{1.5}{100}$$

$$= 0.985\ m^3$$

(b) Volume of Cement = $\dfrac{W_c}{S_c \times 1000} = \dfrac{425}{3.15 \times 1000} = 0.135\ m^3$

(c) Volume of Water = $\dfrac{W_w}{1000} = \dfrac{170}{1000} = 0.170\ m^3$

(d) Volume of Admixture = $\dfrac{W_{adm}}{S_{adm} \times 1000} = \dfrac{1\%\ W_c}{S_{adm} \times 1000}$

$$= \frac{1 \times \frac{425}{100}}{1.145 \times 1000}$$

$$= 0.0037\ m^3.$$

(e) Volume of Total Aggregate = (a) − (b + c + d)

$$= 0.985 - (0.135 + 0.170 + 0.0037)$$

$$= 0.6763\ m^3$$

(f) Weight of C.A = W_{CA} = proportion × (e) × S_{CA} × 1000

$= 0.62 \times 0.6763 \times 2.74 \times 1000$

$W_{CA} = 1148.89$ kg

(g) Weight of F.A. = W_{FA} = [1 − proportion] × (e) × S_{FA} × 1000

$= [1 - 0.62] \times 0.6763 \times 2.74 \times 1000$

$W_{FA} = 704.16$ kg.

⇒ Final Result :-

	C	:	FA	:	CA	:	W	:	Adm.
1 m^3	425	:	704.16	:	1148.89	:	170	:	4.25
Ratio	1	:	1.65	:	2.70	:	0.4	:	0.01
Per bag of cement	50	:	82.5	:	135	:	20	:	0.5

Ex. Calculate standard deviation for the data given below.

Sample No.	Cube 1	Cube 2	Cube 3	Comp. strength of sample (x_i)	$(\bar{x}-x_i)$	$(\bar{x}-x_i)^2$
1	20.5	24.0	22.5	22.3	−2.1	4.41
2	18.5	22.5	19.0	20.0	0.2	0.04
3	19.5	20.5	21.5	20.5	−0.3	0.09
4	22.0	23.0	21.5	22.2	−2.0	4.0
5	18.5	21.5	21.5	20.5	−0.3	0.09
6	22.5	23.5	23.0	23.0	−2.8	7.84
7	24.0	23.5	21.5	23.0	−2.8	7.84
8	22.0	18.5	19.5	20.0	0.2	0.04
9	16.5	15.5	14.0	15.3	4.9	24.04
10	13.0	15.0	17.0	15.0	5.2	27.04
				$\bar{x} = 20.2$		$\Sigma = 75.4$

Standard deviation,

$$S = \sqrt{\frac{\Sigma(\bar{x}-x_i)^2}{(n-1)}}$$

$$= \sqrt{\frac{75.4}{(10-1)}} = 2.85$$

$\Rightarrow \boxed{S = 2.85}$

* Note:
Test data of minimum 30 samples is required to calculate standard deviation.

1.3 Reinforcement:

Any material that can take tension may be used as reinforcement. For e.g. steel, copper, aluminium, plastic fibre, bamboo etc. Steel is most preferable because of following reasons –

i) Economical than other metals.

ii) High tensile strength

iii) Coefficient of thermal expansion is comparable to as of concrete.

1.3.1 Size of Reinforcement:

Diameter	Actual Area	Approx Area.
6	20.27	30
8	50.26	50
10	78.54	75
12	113.09	110
16	201.06	200
20	314.15	314
25	490.87	490 or 500
28	615.75	600
32	804.24	800
36	1017.87	1000
40	1256.63	1250
45	1590.43	1600
50	1963.49	2000

Ex. Provide 20mm and 12mm dia. bars for steel area $1285\,mm^2$

⇒ $A_{st} = 1285\,mm^2$

1. $4-20\phi + 1-12\phi$
2. $3-20\phi + 4-12\phi$

Ex. What will be the dia of bars if 3 bars are allowed for $A_{st} = 1450\,mm^2$

⇒ $$\text{Area of 1-bar} = \frac{1450}{3} = 483.33\,mm^2$$

1. $3-25\phi$
2. $2-28\phi + 1-20\phi$

1.3.2 Grade of Steel:

Yield Stress

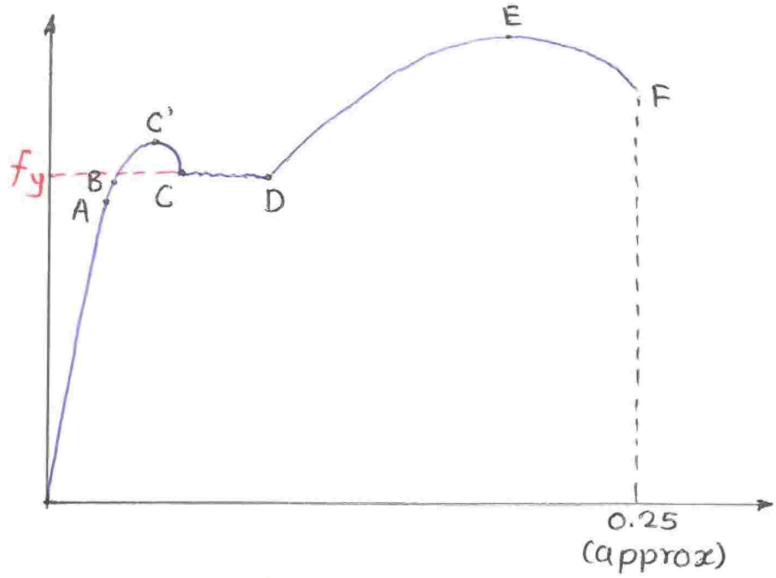
Fe 250 → Mild Steel

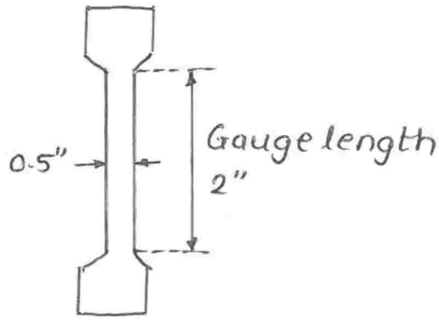
Fe 415
Fe 500 } HYSD

Recent Development {
Fe 500D → Ductility.
Fe 550
Fe 600

1.3.3 Types of Steel:

1. Mild Steel.





OA ⟶ Linear

A ⟶ Proportional Limit

AB ⟶ Non-linear

B ⟶ Elastic Limit

C' ⟶ Upper Yield Point

C ⟶ Lower Yield Point (Yield Stress)

CD ⟶ Yield Plateau

DE ⟶ Strain Hardening

E ⟶ Ultimate Point

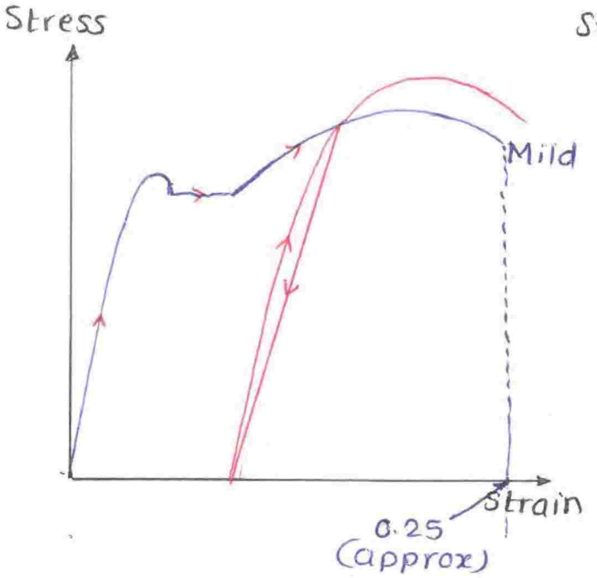
F ⟶ Fracture Point

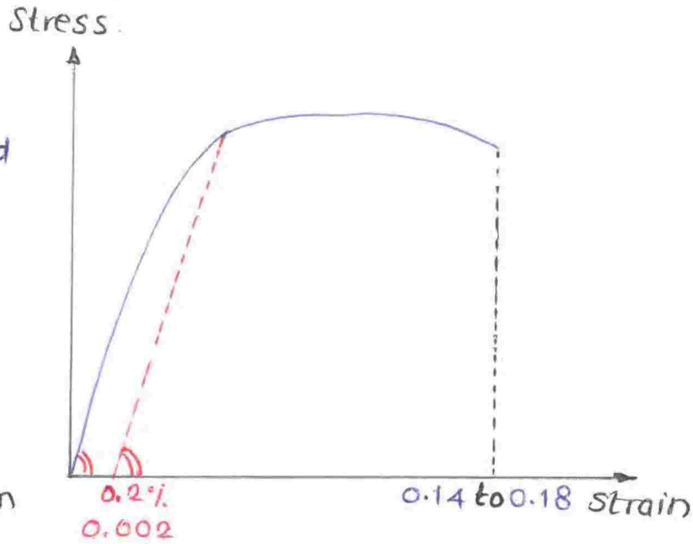
2. HYSD/CTD/TOR

HYSD ⟶ High Yield Strength Deformed

CTD ⟶ Cold Twisted Deformed.

TOR ⟶ (Name of Company)

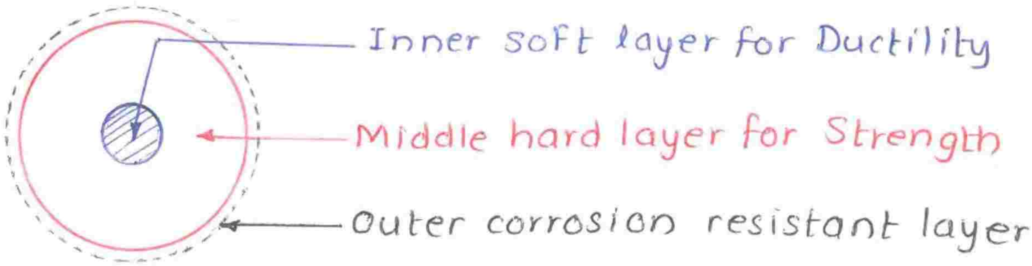




- It is obtained by straining mild steel beyond yield plateau by stretching and twisting at particular temperature and after that unloading is done. Above process of straining eliminates yield plateau from stress-strain diagram.
- HYSD steel is less ductile than mild steel.
- In absence of definite yield point, 0.2% proof stress is considered as yield stress.

3. Thermo Mechanically Treated Bars (TMT)

- These bars are made by applying thermal and mechanical process simultaneously on steel.
- This combination process makes the steel relatively more corrosion resistant.
- TMT bars are relatively more ductile than HYSD.
- TMT bars have all relevent features of HYSD.



TMT Bar

* Note:

- As carbon % increases, strength increases and ductility decreases.

2. Limit State Method

2.1 Introduction:

In this method of design, a structure shall be designed to withstand safely all loads liable to act on it through its life. It shall also satisfy all serviceability requirements such as limitation on deflection and cracking. This acceptance limit for safety and serviceability requirement before failure occurs is Limit State.

2.2 Types of Limit State:

A) Limit State of Collapse:

1. Flexure
2. Shear
3. Torsion
4. Compression.

B) Limit State of Serviceability:

1. Deflection
2. Cracking
3. Vibration

2.3 Design Strength of Material:

This is the strength which is considered for design of structure.

1) Concrete:

$$\text{Design strength} = \frac{\text{Characteristic Strength}}{\text{Partial F.O.S.}}$$

$$= \frac{0.67 f_{ck}}{1.5}$$

$$\approx 0.45 f_{ck}$$

- Lab, Cube, uniaxial loading ⟶ f_{ck}
- Lab, beam, any loading ⟶ $0.67 f_{ck}$
- Site, beam, any loading ⟶ $0.45 f_{ck}$

2) Steel:

$$\text{Design Strength} = \frac{\text{Characteristic Strength}}{\text{Partial F.O.S.}}$$

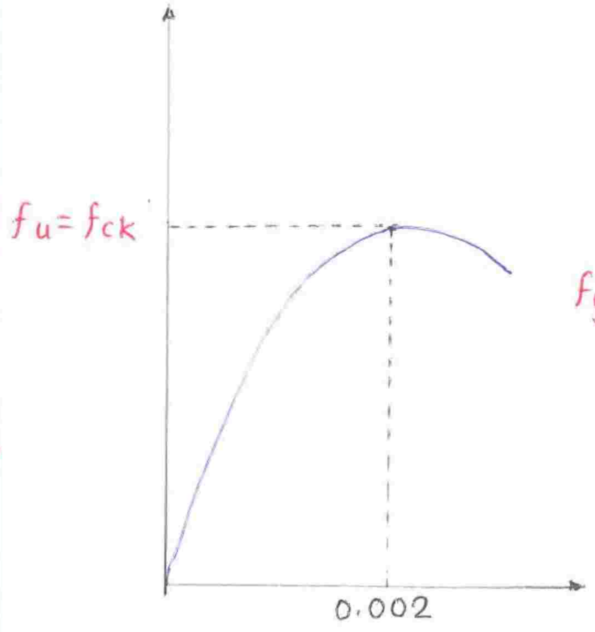
$$= \frac{f_y}{1.15}$$

$$\approx 0.87 f_y$$

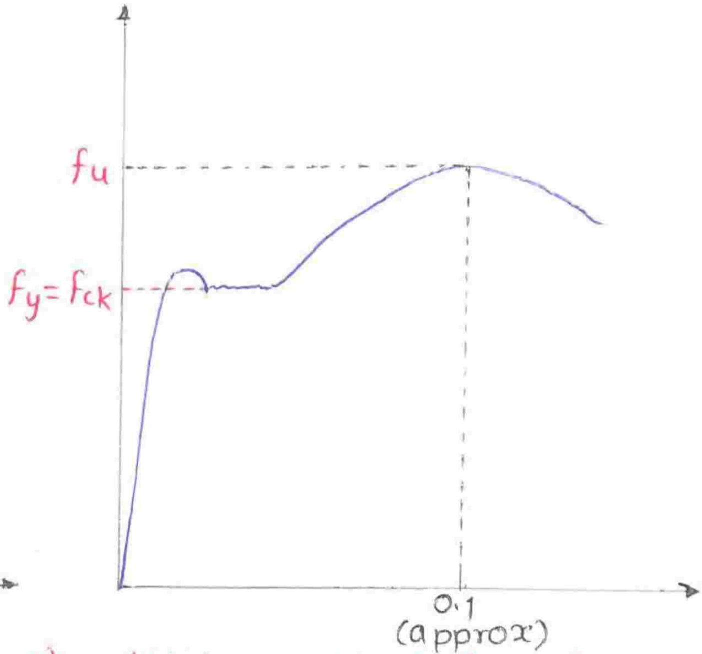
- Lab, bars, tension $\longrightarrow f_y$
- Factory, bars, tension $\longrightarrow 0.87 f_y$

*** Note:**

- Partial F.O.S. with material is considered to account for variation in quality control
- Partial F.O.S is higher than as of steel because quality control of concret is inferior than steel.
- Ultimate strength of concrete is considered as characteristic strength for concrete while yield stress of steel is taken as characteristic strength for stee. This is because ultimate strength is obtained at very high strain which is not possible in Civil engineering structures.



(Possible in Civil Structures)



(Not possible in Civil Structures)

2.4 Design Load:

It is the load for which structure is designed.

Design Load = Characteristic Load × Partial F.O.S.

*** Characteristic load:**

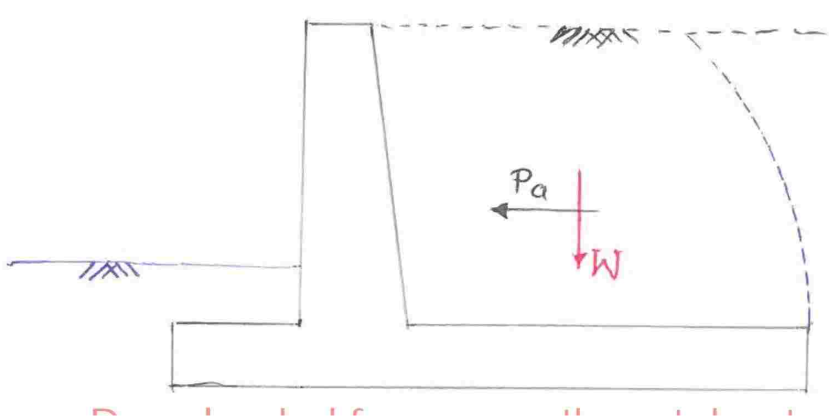
It means that value of load which has 95% probability of not being exceeded during life of structure.

*** Partial F.O.S.**

Load Combinations	Limit State of Collapse			Limit State of Serviceability		
	DL	LL	WL/EL	DL	LL	WL/EL
DL + LL	1.5	1.5	-	1.0	1.0	-
DL + WL/EL	1.5/0.9	-	1.5	1.0	-	1.0
DL + LL + WL/EL	1.2	1.2	1.2	1.0	0.8	0.8

*** Note:-**

- Design wind load and design earthquake load are never considered on a structure simultaneously.
- Partial F.O.S. of Limit State of Collapse is higher than as of Limit State of Serviceability because safety is more important than serviceability.
- Factor 0.9 is taken with DL if DL is providing stability against sliding and overturning.



$$\frac{\text{Restoring Force/Moment}}{\text{Disturbing Force/Moment}} \geq 1.4$$

- If retaining wall is safe in sliding and overturning with 90% of its weight then it must be more safe with 100% weight.

Ex. Calculate Design moment of beam which is subjected to moment due to different loading as given below.

DL ⟶ 100 kN m

LL ⟶ 50 kN·m

WL ⟶ 70 kN·m

EL ⟶ 200 kN·m

⇒ Combinations:

1) DL + LL = 1.5×100 + 1.5×50 = 225 kN·m

2) DL + EL = 1.5×100 + 1.5×200 = 450 kN·m

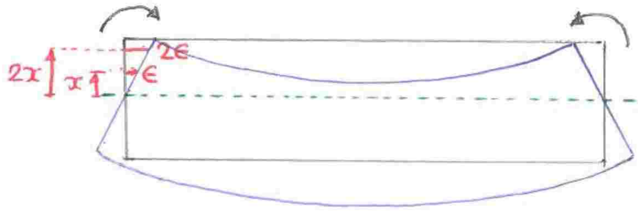
3) DL + LL + EL = 1.2×100 + 1.2×50 + 1.2×200 = 420 kN·m

So, design BM is 450 kN m and critical load combination is DL+EL

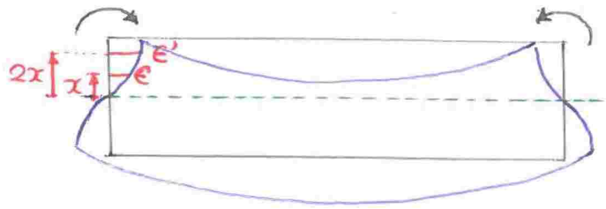
2.5 Limit State of Collapse of Flexure:

2.5.1 Assumptions:

1) Plane section remains plane after bending. It means strain variation over cross-section is linear.



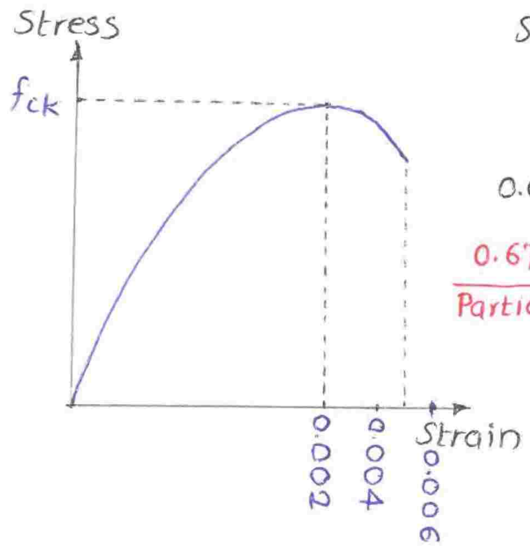
(a) Plane section remains plane after bending

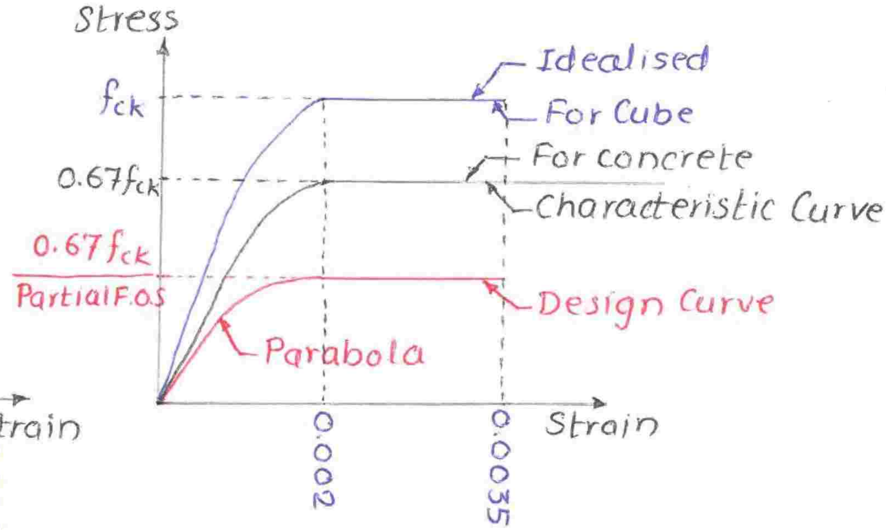


(b) Plane section no longer remains plane after bending.

2) Maximum compressive strain in concrete is limited to 0.0035

3) Stress-strain diagram of concrete is as given below.





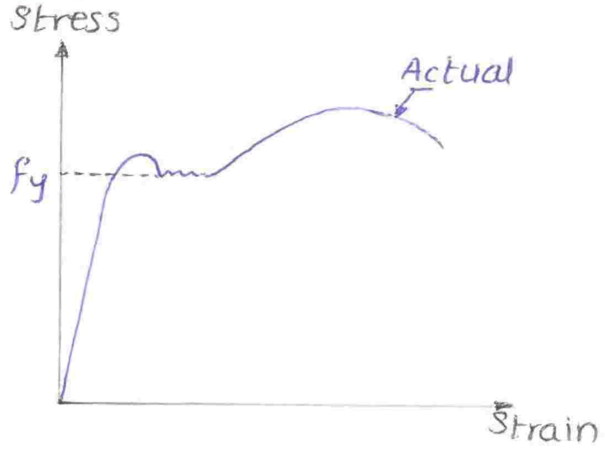
* Note:

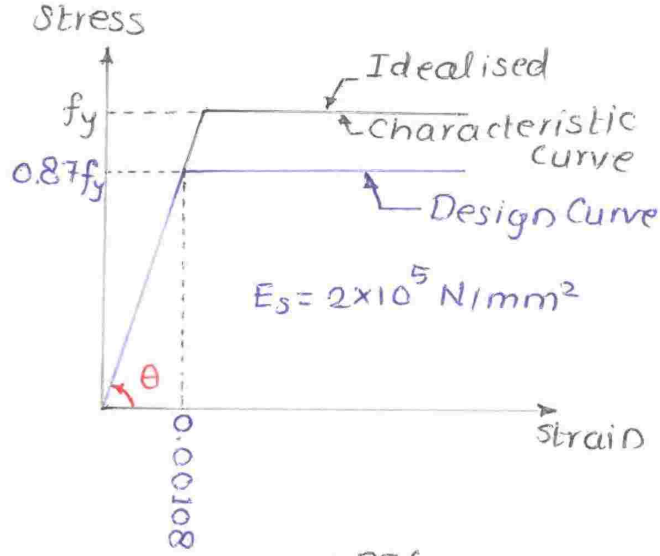
Any variation of stress-strain curve of concrete can be considered provided that must comply with experimental results.

4) Tensile strength of concrete is ignored.

5) Stress-strain diagram of Steel under compression and tension is as given below.

• Mild:

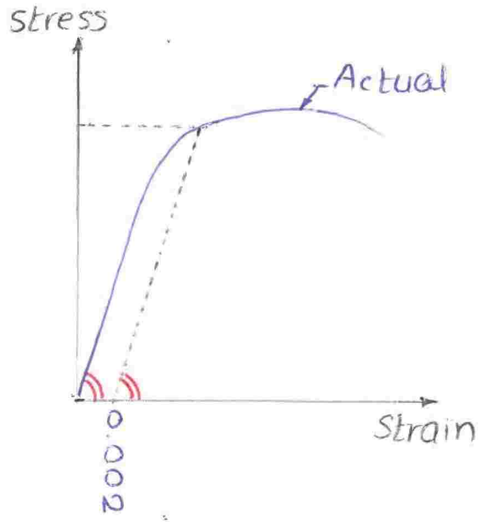


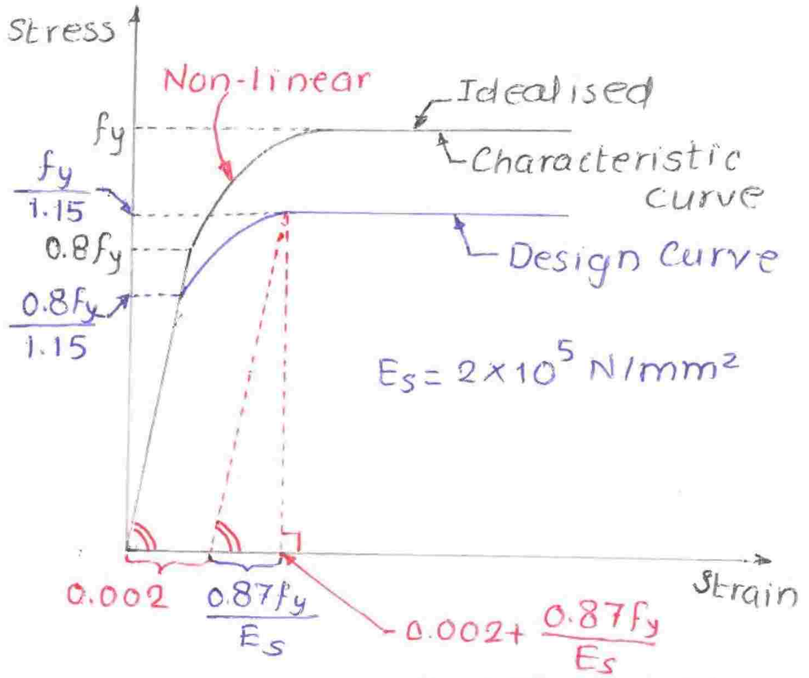


$$\tan\theta = \frac{0.87 f_y}{\text{strain}}$$

$$\Rightarrow \text{Strain} = \frac{0.87 f_y}{E_s} = 0.00108$$

• HYSD:





$$0.002 + \frac{0.87 f_y}{E_s}$$

$= 0.0038$ (Fe415)

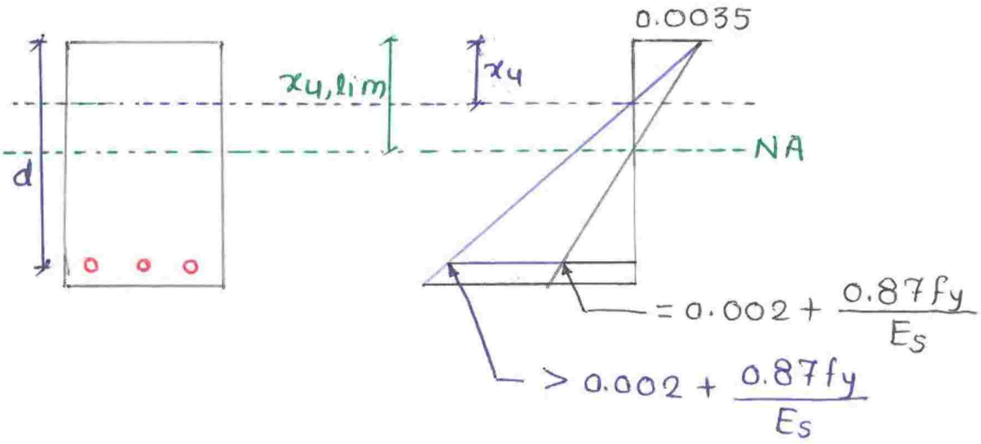
$= 0.0042$ (Fe500)

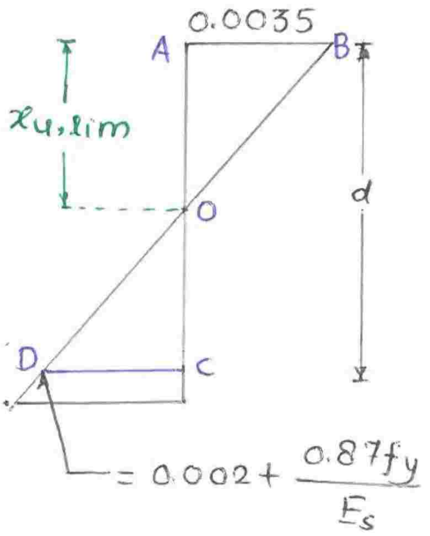
6) Maximum strain in tension steel at the time of failure should not be less than $\left(0.002 + \frac{0.87 f_y}{E_s}\right)$

* Note:-

- Maximum limit of maximum strain of tension steel at the time of failure is not defined because steel is very ductile as compared to concrete.
- Minimum limit of maximum strain of tension steel at the time of failure $\left(0.002 + \frac{0.87 f_y}{E_s}\right)$ is defined for full utilization of strength of steel $(0.87 f_y)$

Based on above assumptions:





from similar triangle

$$\frac{OA}{AB} = \frac{OC}{CD}$$

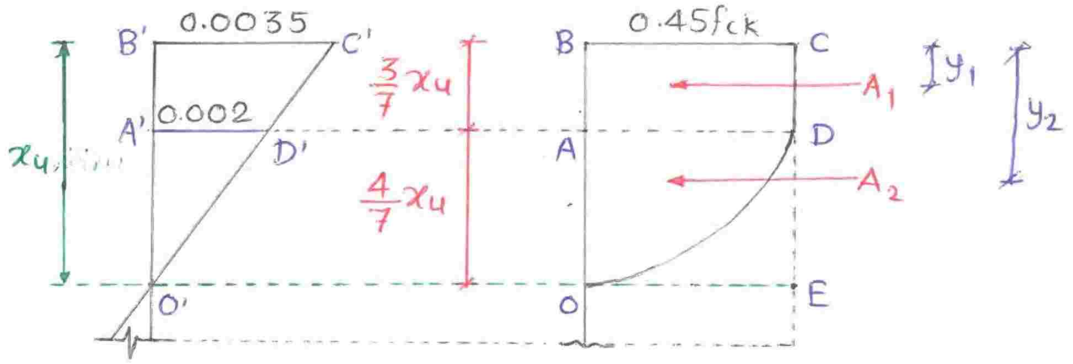
$$\Rightarrow \frac{x_{u,lim}}{0.0035} = \frac{d - x_{u,lim}}{0.002 + \frac{0.87 f_y}{E_s}}$$

$x_{u,lim} = 0.53d$ (Fe 250)

$= 0.48d$ (Fe 415)

$= 0.46d$ (Fe 500)

2.5.2 Analysis of Stress Block:



Strain   Stress Block

From strain diagram,

$$\frac{O'A'}{A'D'} = \frac{O'B'}{B'C'}$$

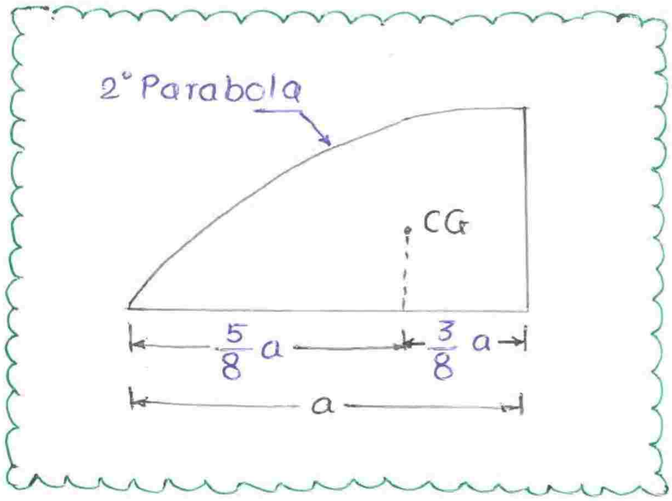
$$\frac{O'A'}{0.002} = \frac{x_u}{0.0035}$$

$$O'A' = \frac{4}{7}x_u$$

Now, $A'B' = O'B' - O'A'$

$$= x_u - \frac{4}{7}x_u$$

$$A'B' = \frac{3}{7}x_u$$



Area of Stress Block = Area of OADO + Area of ABCDA

$$= \frac{2}{3} \times \text{Area of OADEO} + \text{Area of ABCDA}$$

$$= \frac{2}{3} \times \frac{4}{7}x_u \times 0.45 f_{ck} + \frac{3}{7}x_u \times 0.45 f_{ck}$$

Area of Stress Block $= 0.36 f_{ck} x_u$

Position of CG. from top fibre $= \dfrac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$

$$= \frac{\left(\frac{3}{7}x_u \times 0.45 f_{ck}\right) \times \left(\frac{1}{2} \times \frac{3}{7}x_u\right) + \left(\frac{2}{3} \times \frac{4}{7}x_u \times 0.45 f_{ck}\right) \times \left(\frac{3}{7}x + \frac{3}{8} \times \frac{4}{7}x_u\right)}{0.36 f_{ck} x_u}$$

Position of C.G from top fibre $= 0.42 x_u$

Ex. Calculate total probability of failure of RCC structure as per IS456

⇒

Total Probability of failure = Prob. of Material strength below ch. strength × Prob. of load below ch. load + Prob. of material strength above ch. strength × Prob. of load above ch. load + Prob. of material strength below ch. strength × Prob. of load above ch. load

$= (0.05 \times 0.95) + (0.95 \times 0.05) + (0.05 \times 0.05)$

Total Probability of failure $= 0.0975$

$= 9.75 \times 10^{-2}$

[order ⇒ -2]

134.23×10^{3}

1.3423×10^{5} → order

only one digit

2.6 Important Points of LSM:

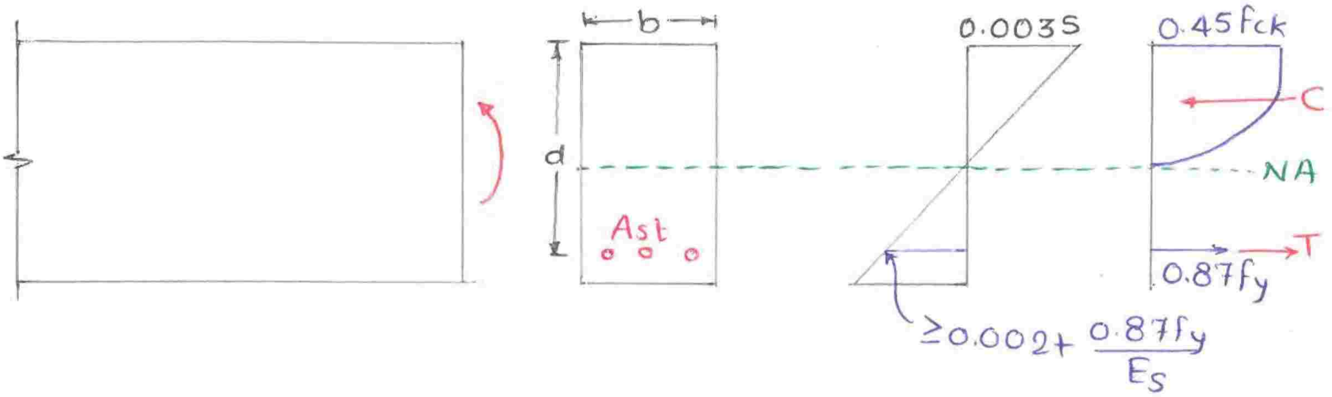
- Failure is based on principal strain theory
- Safety is checked at ultimate load while serviceability is checked at service/working load.
- LSM gives a smaller section dimension with more amount of reinforcement as compared to WSM.
- LSM is economical than WSM
- Design load in LSM is ultimate load (char. load × Partial F.O.S) while characteristic load is considered as design load for WSM.

3. Singly Reinforced Rectangular Section

3.1 Introduction:

If reinforcement is provided in tension zone only then section is classified as single reinforced.

3.2 Position of Neutral Axis:



For position of N.A.

$$C = T$$

$$0.36\, f_{ck}\, x_u \cdot b = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36\, f_{ck} \cdot b}$$

$dC = dx \cdot dy \cdot \sigma$

$C = \int dx \cdot dy \cdot \sigma$

= Volume of Shape

$C = \text{Area of stress block} \times b$

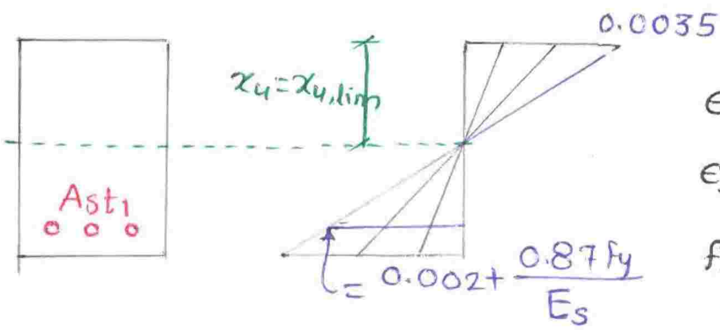
From above expression, it is clear that position of N.A is directly proportional to the amount of steel in tension.

3.3 Types of Section:

Based on amount of tension steel in section, three types of section are defined.

1) Balanced Section:

Amount of steel in section is such that strain in concrete is 0.0035 and strain in tension steel is $0.002 + \frac{0.87 f_y}{E_s}$, at the time of failure.



$\epsilon_c = 0.0035 \qquad f_c = 0.45 f_{ck}$

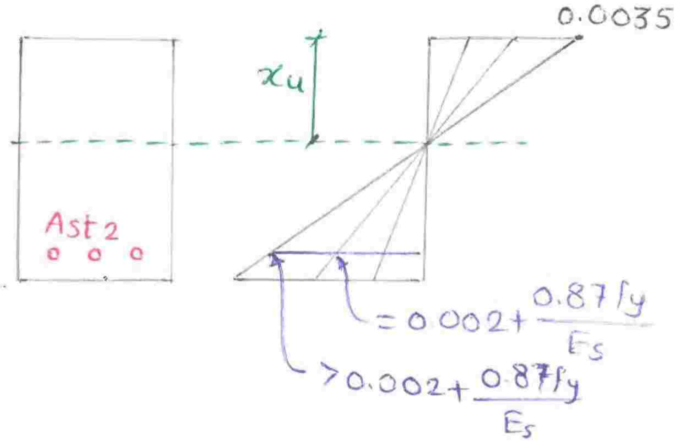
$\epsilon_s = 0.002 + \frac{0.87 f_y}{E_s}$

$f_s = 0.87 f_y$

$x_u = x_{u,lim}$

2) Under Reinforced Section:

Amount of steel in section is such that strain in concrete is 0.0035 and strain in tension steel is greater than $0.002 + \frac{0.87 f_y}{E_s}$, at the time of failure.



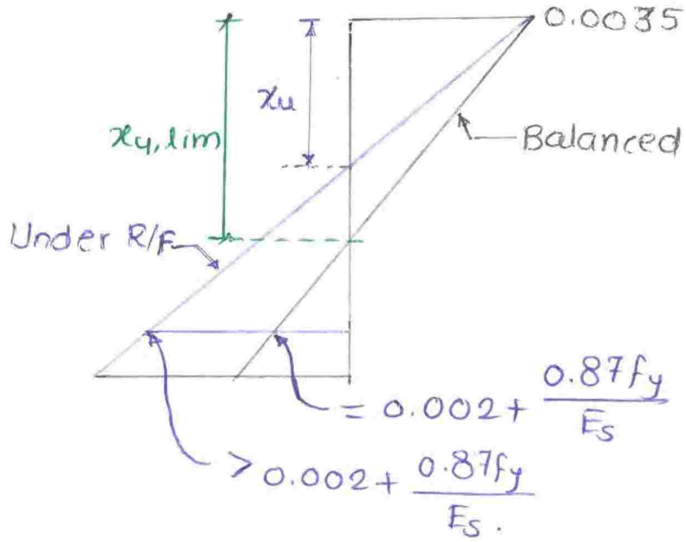
$\epsilon_c = 0.0035 \quad f_c = 0.45 f_{ck}$

$\epsilon_s > 0.002 + \frac{0.87 f_y}{E_s}$

$f_s = 0.87 f_y$

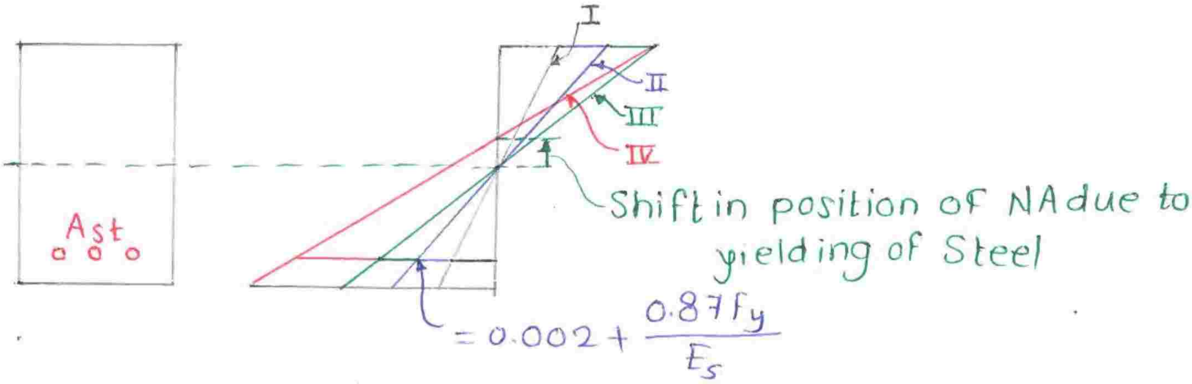
$x_u < x_{u,lim}$

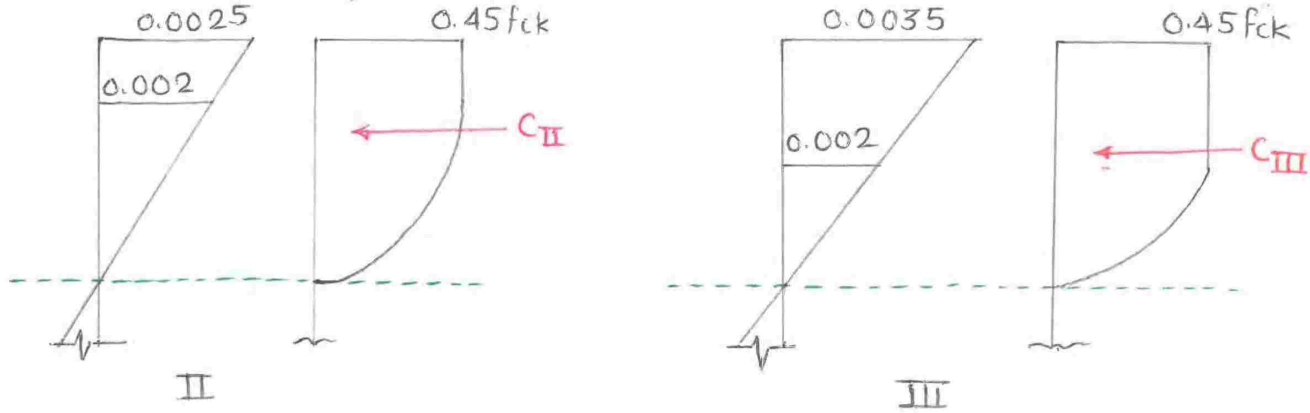
* Comparing failure strain profile of balanced and under reinforced section:



$x_u < x_{u,lim}$

* Shift in position of N.A. due to yielding of Steel:



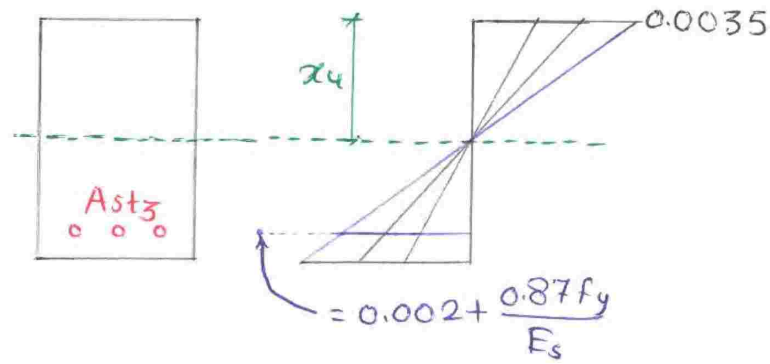


$C_{III} > C_{II}$

- For strain profile II and III net tensile force is same because stress in tension steel remains constant ($0.87 f_y$)
- Compressive force for strain profile III is more than compressive force for strain profile II
- To maintain tensile force equal to compressive force, NA shifts upward and final strain profile becomes IV

3> Over Reinforced Section

Amount of steel in section is such that strain in concrete is 0.0035 and strain in tension steel is less than $0.002 + \frac{0.87 f_y}{E_s}$, at the time of failure.



$\epsilon_c = 0.0035$

$f_c = 0.45 f_{ck}$

$\epsilon_s < 0.002 + \frac{0.87 f_y}{E_s}$

$f_s < 0.87 f_y$

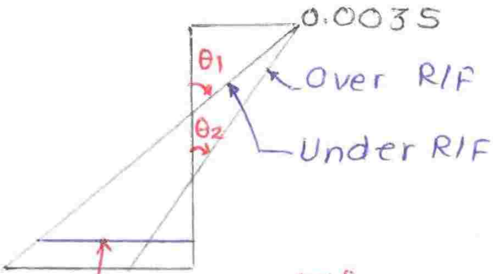
$x_u > x_{u,lim}$ ... (by compairing failure strain profile of balanced & over reinforced section)

* Note:

- Section always fails due to crushing of concrete in LSM.
- Steel never fails (fracture) it always yields.
- Types of failure;
  - i) Under R/F ⟶ Tension failure
    ⟶ Secondary comp. failure
  - ii) Over R/F ⟶ Compression failure
    ⟶ Primary comp. failure

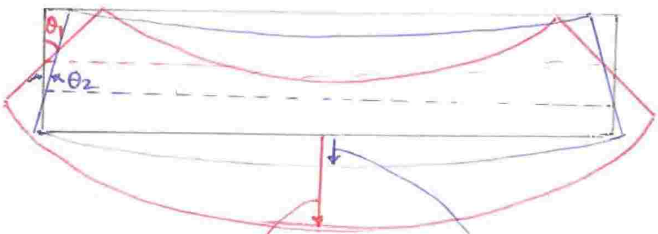
- Over-reinforced sections are not permitted because failure is brittle and sudden (without warning)

- Under reinforced sections are preferable because they give sufficient warning before failure and show ductile failure.



$= 0.002 + \frac{0.87 f_y}{E_s}$

$\theta_1 > \theta_2$

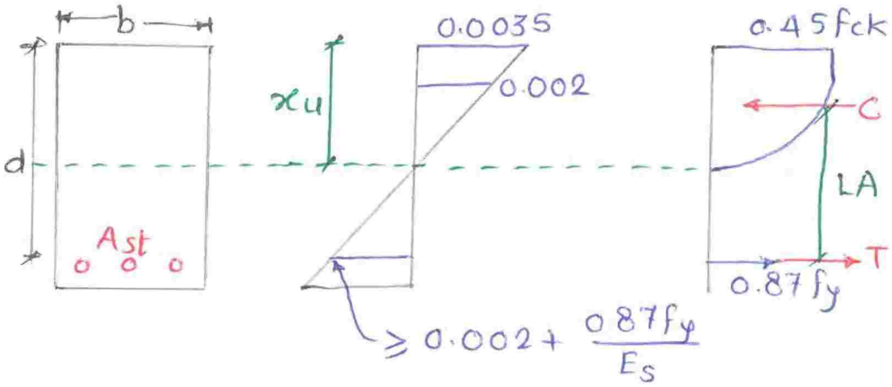


$\Delta_{Under\ R/F} > \Delta_{over\ R/F}$

- In under reinforced section, deflection and cracking are more than over reinforced section.

- Practically, all flexure sections are under reinforced in LSM.

3.4 Moment of Resistance of Section:



$MR = C \times LA$ & $T \times LA$

1) Balanced Section:

$$MR = C \times LA$$
$$= 0.36\, f_{ck}\, x_{u,lim} \cdot b\, (d - 0.42\, x_{u,lim})$$
$$= 0.148\, f_{ck}\, bd^2 \quad (Fe\ 250)$$
$$= 0.138\, f_{ck}\, bd^2 \quad (Fe\ 415)$$
$$= 0.133\, f_{ck}\, bd^2 \quad (Fe\ 500)$$

$$MR = Q\, bd^2 = M_{u,lim}$$

$$MR = T \times LA$$
$$M_{u,lim} = 0.87\, f_y \cdot A_{st} \cdot (d - 0.42\, x_{u,lim})$$

For A_{st} of balanced section:

$$C = T$$
$$0.36\, f_{ck}\, x_{u,lim} \cdot b = 0.87\, f_y \cdot A_{st,lim}$$
$$A_{st,lim} = 0.414 \left(\frac{f_{ck}}{f_y}\right) x_{u,lim} \cdot b$$

2) Under Reinforced Section:

$MR = C \times LA \Rightarrow MR = 0.36\, f_{ck}\, x_u \cdot b\, (d - 0.42\, x_u)$

$MR = T \times LA \Rightarrow MR = 0.87\, f_y \cdot A_{st}\, (d - 0.42\, x_u)$

3> Over Reinforced Section:

$MR = C \times LA$

$MR = 0.36 f_{ck} x_u \cdot b \, (d - 0.42 x_u)$

$MR = T \times LA$

$MR = f_{st} \cdot A_{st} \cdot (d - 0.42 x_u)$

where,

$f_{st} < 0.87 f_y$

3.5 Types of Problem:

A> Analysis:

1. Position of NA.
2. MR of Section
3. Ast required for balanced Section.

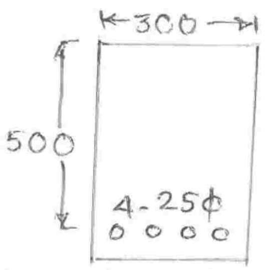
B> Design:

1. Section size is given and Ast is to be calculated
2. Section size and Ast both are to be calculated.

3.5.1 Position of NA:

Always equate net compressive force and net tensile force of section to get x_u and compare it with $x_{u,lim}$.

Ex. Determine the position of N.A. for the given section.



M20
Fe250

$\Rightarrow$ For Position of NA. $\Rightarrow C = T \Rightarrow 0.36 f_{ck} x_u \cdot b = 0.87 f_y \cdot A_{st}$

$$0.36 \times 20 \times x_u \times 300 = 0.87 \times 250 \times 4 \times \frac{\pi}{4} \times 25^2$$

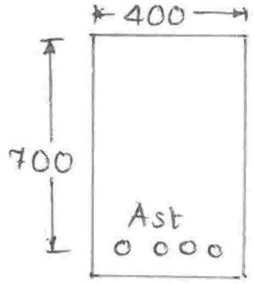
$$x_u = 197.71 \text{ mm}$$

Now, $x_{u,lim} = 0.53d = 0.53 \times 500 = 265 \text{ mm}$

3.5.2 Moment of Resistance of Section:

Use procedure as illustrated in section 3.4

Ex. Calculate MR. of given section



M25
Fe415

i) $A_{st} = 3 - 25\phi$

ii) $A_{st} = 8 - 25\phi$

iii) A_{st} for balanced section.

$\Rightarrow$

i) $A_{st} = 3 - 25\phi$

For position of NA.

$$C = T$$

$$0.36 f_{ck} x_u \cdot b = 0.87 f_y \cdot A_{st}$$

$$0.36 \times 25 \times x_u \times 400 = 0.87 \times 415 \times 3 \times \frac{\pi}{4} \times 25^2$$

$$x_u = 147.65 mm$$

Now,

$$x_{u,lim} = 0.48 d$$

$$= 0.48 \times 700$$

$$x_{u,lim} = 336 \text{ mm}$$

Since, $x_u < x_{u,lim}$ so section is under reinforced

Moment of Resistance:

From compression side,

$$MR = C \times LA$$

$$= 0.36 f_{ck} x_u \cdot b \cdot (d - 0.42 x_u)$$

$$= 0.36 \times 25 \times 147.65 \times 400 \times (700 - 0.42 \times 147.65)$$

$$MR = 339.19 \text{ kN m}$$

From tension side,

$$MR = T \times LA = 0.87 f_y \cdot A_{st} \cdot (d - 0.42 x_u)$$

$$= 0.87 \times 415 \times 3 \times \frac{\pi}{4} \times 25^2 \times (700 - 0.42 \times 147.65)$$

$$MR = 339.19 \text{ kN.m}.$$

ii) $A_{st} = 8\text{-}25\phi$

For position of NA.

$$C = T$$

$$0.36 f_{ck} x_u \cdot b = 0.87 f_y \cdot A_{st}$$

$$0.36 \times 25 \times x_u \times 400 = 0.87 \times 415 \times 8 \times \frac{\pi}{4} \times 25^2$$

$$x_u = 393.84 \text{ mm}$$

Since $x_u > x_{u,lim}$ so section is over reinforced.

Exact position of NA for this section will lie between $x_{u,lim}$ (336 mm) and 393.84 mm because stress in tension steel will be less than $0.87 f_y$.

Since over reinforced sections are not permitted so we are not intrested in calculation of exact position of NA and MR of section.

iii) A_{st} for balanced section.

$$A_{st} = 0.414 \left(\frac{f_{ck}}{f_y}\right) x_{u,lim} \cdot b$$

$$= 0.414 \times \frac{25}{415} \times 336 \times 400$$

$$A_{st,lim} = 3351.90 \text{ mm}^2.$$

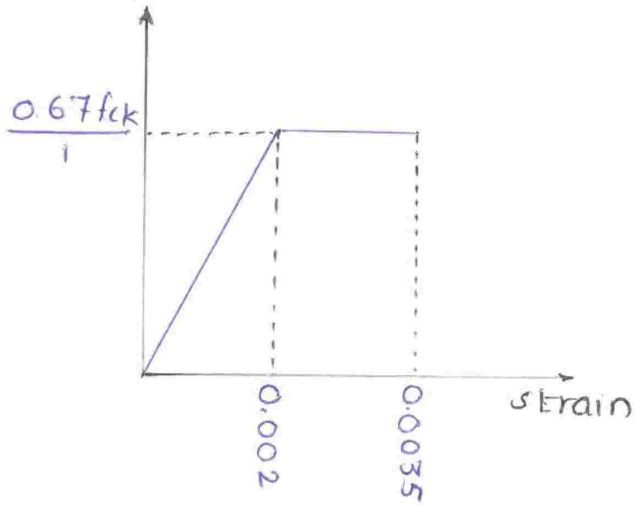
Ex. A rectangular under reinforced section of effective size (300 x 500) mm is reinforced with $3\text{-}16\phi$ of Fe 415. M20 concrete

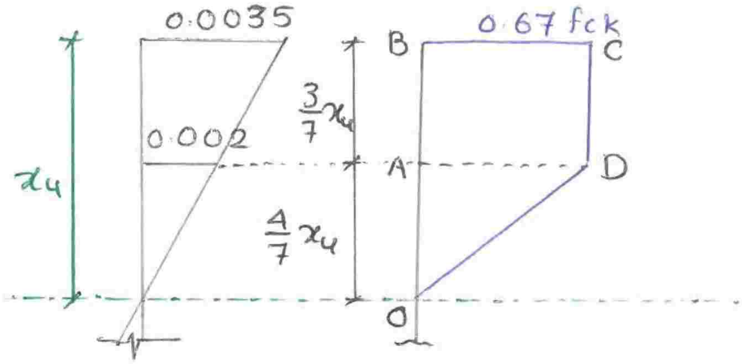
Assume straight line instead of parabolla for stre-strain curve of concrete and partial F.O.S. = 1

a) Calculate position of N.A. w.r.t. extreme comp. fibre

b) Calculate difference between depth of N.A. calculated as per IS456 and part (a) of this problem.

⇒





Area of Stress block $= A_{OBCDO} = \frac{1}{2}(OB + CD)\,BC$

$$= \frac{1}{2}\left(x_u + \frac{3}{7}x_u\right) \cdot 0.67\, f_{ck}$$

Area of stress block $= 0.4785\, f_{ck}\, x_u$

a) Position of NA·

$$C = T$$

$$0.4785\, f_{ck}\, x_u \cdot b = 0.87\, f_y \cdot A_{st}$$

$$0.4785 \times 20 \times x_u\, 300 = 0.87 \times 415 \times 3 \times \frac{\pi}{4} \times 16^2$$

$\Rightarrow x_u = 75.85$ mm.

b) Position of NA as per IS 456

$$C = T$$

$$0.36\, f_{ck}\, x_u \cdot b = 0.87\, f_y \cdot A_{st}$$

$$0.36 \times 20 \times x_u \times 300 = 0.87 \times 415 \times 3 \times \frac{\pi}{4} \times 16^2$$

$\Rightarrow x_u = 100.82$ mm.

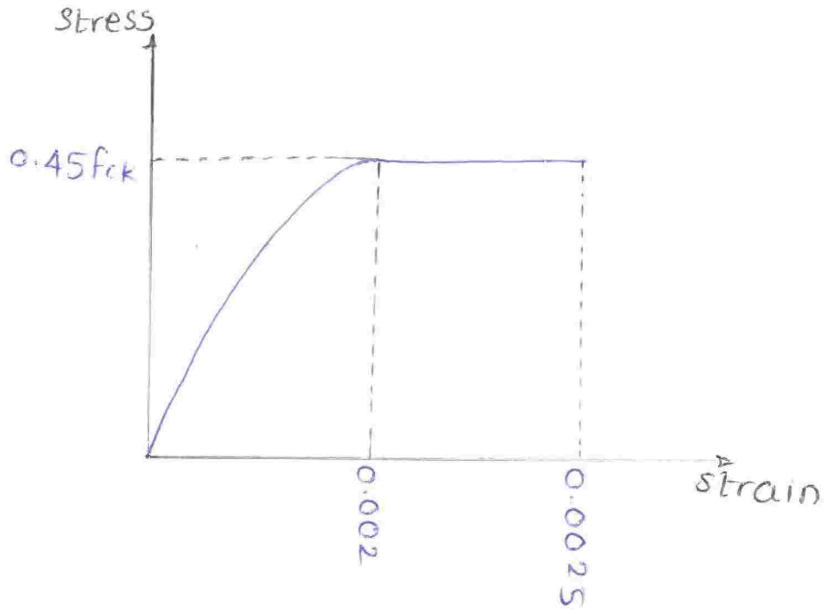
$\Rightarrow$ So, difference $= 100.82 - 75.85 \simeq 25$ mm.

Ex. In the design of beam by LSM in flexure as per IS 456, let the maximum strain limited to 0.0025. For this situation, consider a rectangular section of effective size (250 × 350) mm. $A_{st} = 1500\ mm^2$, Fe 250 and M30

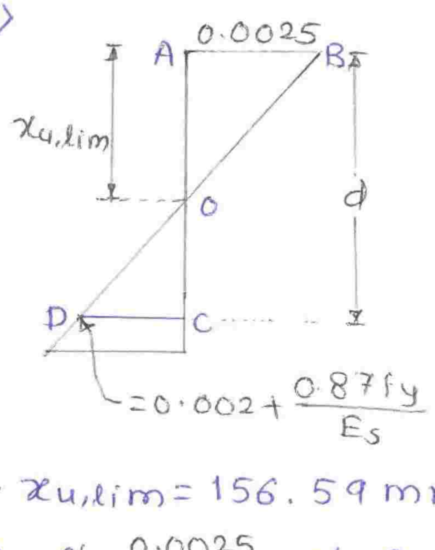
a) Position of NA for balanced section

b) At the limit stat of collapse of flexure, calculate force acting on compression zone of section.

⇒



a)



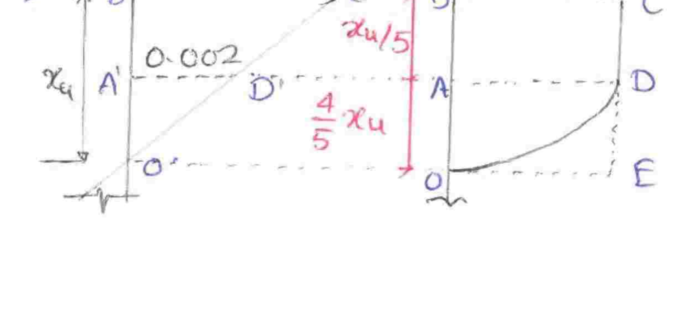
from similar triangle

$$\Rightarrow \frac{x_{u,lim}}{0.0025} = \frac{d - x_{u,lim}}{0.002 + \frac{0.87 f_y}{E_s}}$$

$$\Rightarrow \frac{x_{u,lim}}{0.0025} = \frac{350 - x_{u,lim}}{0.002 + \frac{0.87 \times 250}{2 \times 10^5}}$$

$\Rightarrow x_{u,lim} = 156.59$ mm.

b)



from strain diagram

$$\frac{O'A'}{A'D'} = \frac{O'B'}{B'C'}$$

$$\frac{O'A'}{0.002} = \frac{x_u}{0.0025}$$

$$O'A' = \frac{4}{5}x_u$$

Now, $A'B' = O'B' - O'A'$

$$= x_u - \frac{4}{5}x_u$$

$$A'B' = \frac{x_u}{5}$$

Area of stress block $= A_{OADO} + A_{ABCDA}$

$$= \frac{2}{3}A_{OADEO} + A_{ABCDA}$$

$$= \frac{2}{3}\left(\frac{4}{5}\cdot x_u \times 0.45\, f_{ck}\right) + \left(\frac{x_u}{5} \times 0.45\, f_{ck}\right)$$

Area of stress block $= 0.33\, f_{ck}\, x_u$.

For position of N.A.

$$C = T$$

$$0.33\, f_{ck}\, x_u \cdot b = 0.87\, f_y \cdot A_{st}$$

$$0.33 \times 30 \times x_u \times 250 = 0.87 \times 250 \times 1500$$

$$x_u = 131.81\ \text{mm}$$

Since $x_u < x_{u,lim}$, so section is under reinforced

$$C = 0.33\, f_{ck}\, x_u \cdot b$$

$$= 0.33 \times 30 \times 131.81 \times 250$$

$$\Rightarrow C = 326\ \text{kN}$$

3.5.3 Design of Singly Reinforced Rectangular Section.

Case I: Section size is given and Ast is to be calculated:

Step1: Calculate design/factored/Ultimate BM by multiplying partial F.O.S. to service/working bending moment.

Step2: Calculate $M_{u,lim}$ of given section.

$M_{u,lim} = Q b d^2$

Where, $Q = 0.148 f_{ck}$ (Fe 250)

$= 0.138 f_{ck}$ (Fe 415)

$= 0.133 f_{ck}$ (Fe 500)

Step3: If $BM_u > M_{u,lim}$ then doubly reinforced section is designed

If $BM_u < M_{u,lim}$ then singly under reinforced section is designed.

$BM_u = MR$

$BM_u = 0.36 f_{ck} x_u \cdot b \cdot (d - 0.42 x_u)$

$x_u = ??$

For Ast: $C = T$

$0.36 f_{ck} x_u \cdot b = 0.87 f_y \cdot A_{st}$

$A_{st} = ??$

Alternatively,

$$A_{st} = \frac{0.5 f_{ck} b d}{f_y}\left[1 - \sqrt{1 - \frac{4.6 BM_u}{f_{ck} b d^2}}\right]$$

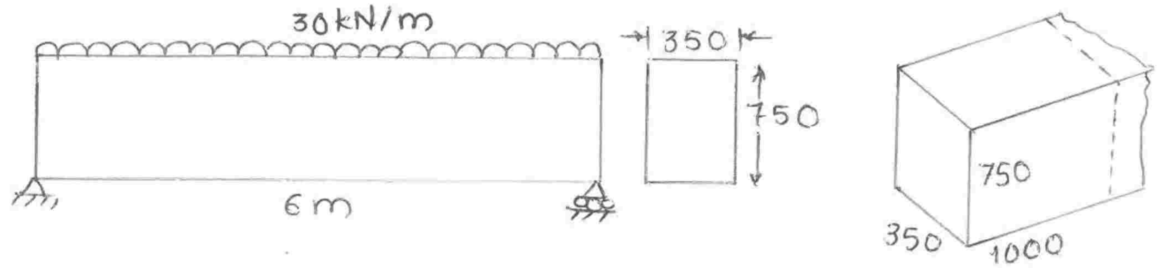
Step4: Ast calculated above should be within permissible limit

$\frac{A_{st,min}}{bd} > \frac{0.85}{f_y}$ ... (To prevent any sudden failure and for ductility)

$A_{st} \leq$ Minimum $\begin{cases} \cdot\ A_{st,lim} & \text{(Over reinforced} \\ \cdot\ 4\% \text{ of gross area} & \text{(Problem in compaction)} \end{cases}$

Ex. Design critical section of RCC beam of overall size (350x750) mm, subjected to live load of 30 kN/m over an effective simply supported span 6 m. Effective cover 50 mm. M30, Fe 415.

⇒



Step 1: Factored/Ultimate BM:

$DL = 0.35 \times 0.75 \times 1 \times 25 = 6.56\ kN/m$

$LL = 30\ kN/m$

Total factored load $= 1.5 \times (6.56 + 30) = 54.84\ kN/m$

$$BM_{u(mid\text{-}span)} = \frac{w_u l^2}{8} = \frac{54.84 \times 6^2}{8}$$

$BM_u = 246.78\ kN{\cdot}m$

Step 2: $M_{u,lim}$:

$$M_{u,lim} = 0.138\, f_{ck}\, b d^2 = 0.138 \times 30 \times 350 \times (750-50)^2$$

$M_{u,lim} = 710.01\ kNm$

Step 3: Since $BM_u < M_{u,lim}$ so section is designed as singly under reinforced section

$BM_u = MR$

$BM_u = 0.36\, f_{ck}\, x_u \cdot b\, (d - 0.42\, x_u)$

$246.78 \times 10^6 = 0.36 \times 30 \times x_u \times 350 \times (700 - 0.42 \cdot x_u)$

$\Rightarrow x_u = 99.16\ mm$

For A_{st} ⇒ $C = T$

$0.36\, f_{ck}\, x_u \cdot b = 0.87\, f_y \cdot A_{st}$

$0.36 \times 30 \times 99.16 \times 350 = 0.87 \times 415 \times A_{st}$ ⇒

Alternatively,

$$A_{st} = \frac{0.5 f_{ck} bd}{f_y}\left[1 - \sqrt{1 - \frac{4.6\, BM_u}{f_{ck} bd^2}}\right]$$

$$= \frac{0.5 \times 30 \times 350 \times 700}{415} \times \left[1 - \sqrt{1 - \frac{4.6 \times 246.78 \times 10^6}{30 \times 350 \times 700^2}}\right]$$

$\Rightarrow A_{st} = 1037.73\ mm^2$

Step 4: Permissible limits:

$$\frac{A_{st,min}}{bd} > \frac{0.85}{f_y}$$

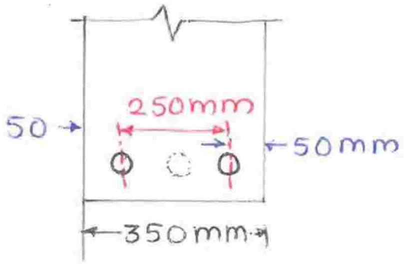
$$A_{st,min} > \frac{0.85 \times 350 \times 700}{415}$$

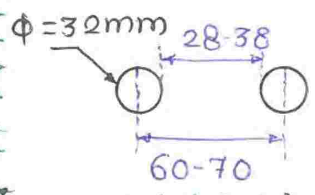
$A_{st,min} > 501.81\ mm^2$

A_{st} < Minimum
- $A_{st,lim} = 0.414\left(\frac{f_{ck}}{f_y}\right) \cdot x_{u,lim} \cdot b$
  $= 0.414 \times \left(\frac{30}{415}\right) \times 99.16 \times 350$
  $= 3519.4\ mm^2$
- $0.04bD = 0.04 \times 350 \times 750 = 10500\ mm^2$

$A_{st} < 3519.4\ mm^2$

In general, 60-70 mm c/c spacing is sufficient for comfortable concreting.





For nominal size of aggregate 20mm
Minimum clear spacing = 20 + 5mm

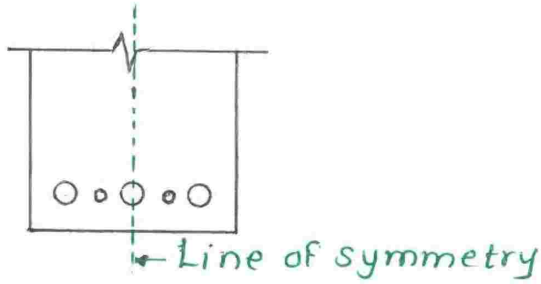
For 3 bars, c/c spacing = 250/2 = 125mm
4 bars, c/c spacing = 250/3 = 83.33 mm
5 bars, c/c spacing = 250/4 = 62.5mm
6 bars, c/c spacing = 250/5 = 50mm (Not preferable)

$A_{st} = 1038\ mm^2$

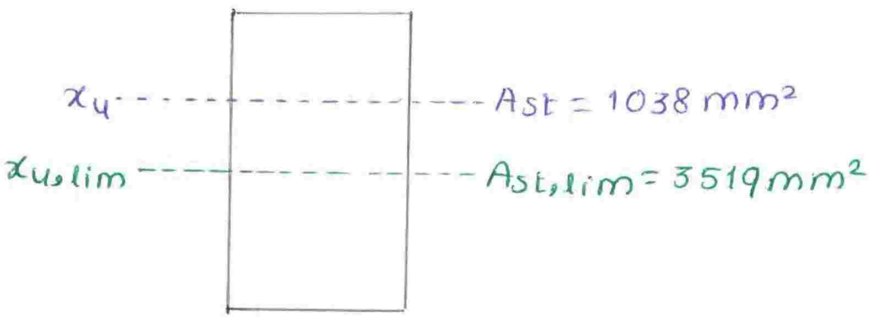
- 2-28ϕ
- 2-25ϕ + 1-12ϕ
- 4-20ϕ
- 2-20ϕ + 3-16ϕ.

* Note:-

- Reinforcement is always provided simmetrically in beam cross-section.



- Section never becomes over reinforced even if more steel is given than calculated, provided given steel is less than $A_{st,lim}$.



Case II: Section size and A_{st} both are to be calculated.

Step 1: Calculate factored/Ultimate/Design BM.

Step 2: Assume suitable value of $\frac{b}{d}$ ratio

(Lateral Buckling) $0.3 < \frac{b}{d} < 0.7$ (Uneconomical)

$\frac{b}{d} = 0.5$ for exam

Step 3: Calculate d required for balanced section

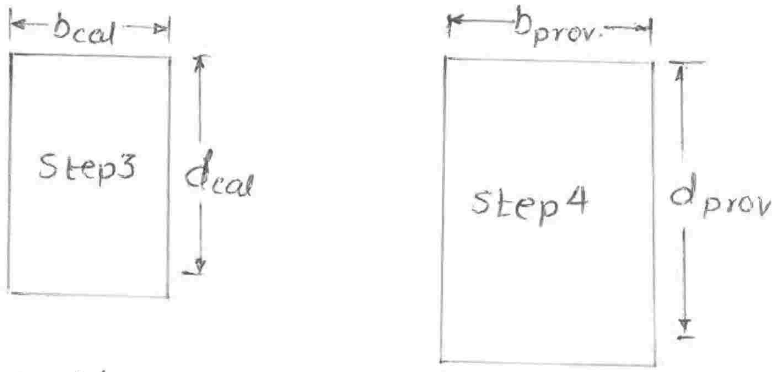
$BM_u = M_{u,lim}$

$BM_u = Q b d^2$

$d = ??$

Step 4: Provide d suitably higher than calculated above and calculate b accordingly

Step 5: Section size provided is step 4 is larger than required in step 3, so provided section is under reinforced.



$BM_u = M_{u,lim}$ $\quad$ $BM_u < M_{u,lim}$

$$A_{st} = \frac{0.5 f_{ck} b d}{f_y}\left[1 - \sqrt{1 - \frac{4.6\, BM_u}{f_{ck} b d^2}}\right]$$

Step 6: A_{st} calculated above should be within permissible limits

Ex. Design a singly reinforced rectangular section for service BM 200 kNm, M30, Fe415, effective cover 50mm

⇒

Step 1: $BM_u = 1.5 \times 200 = 300\,kNm$

Step 2: Assume $\frac{b}{d} = 0.5$

Step 3: 'd' required for balanced section

$$BM_u = M_{u,lim}$$

$$BM_u = 0.138 f_{ck}\, bd^2$$

$$300 \times 10^6 = 0.138 \times 30 \times 0.5 d^3$$

$$d = 525.27\,mm$$

Step 4: Providing, $d = 600\,mm$

$b = 0.5d = 300\,mm$

$D = d + \text{effective cover}$

$= 600 + 50$

$D = 650\,mm$

Step 5: Ast. Required:

$$A_{st} = \frac{0.5 f_{ck} bd}{f_y}\left[1 - \sqrt{1 - \frac{4.6 BM_u}{f_{ck} bd^2}}\right]$$

$$= \frac{0.5 \times 30 \times 300 \times 600}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 300 \times 10^6}{30 \times 300 \times 600^2}}\right]$$

$$A_{st} = 1576.56\,mm^2$$

Step 6: Permissible Limit:

$$\frac{A_{st,min}}{bd} > \frac{0.85}{f_y} \Rightarrow A_{st,min} > \frac{0.85 \times 300 \times 600}{415}$$

$$A_{st,min} > 368.67\,mm^2$$

$$A_{st} \leq \text{Minimum} \begin{cases} \bullet\ A_{st,lim} = 0.414 \left(\dfrac{f_{ck}}{f_y}\right) x_{u,lim}\, b \\ \quad = 0.414 \times \left(\dfrac{30}{415}\right) \times 0.48 \times 600 \times 300 \\ \quad = 2585.75\ mm^2 \\ \bullet\ 0.04bD = 0.04 \times 300 \times 650 = 7800\ mm^2 \end{cases}$$

$A_{st} = 1576.56\ mm^2$

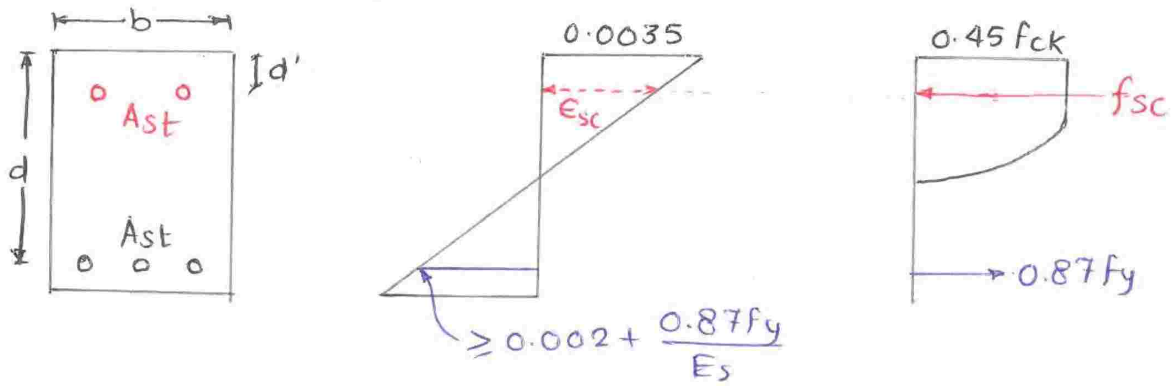
Provide
1. 2-32 ϕ
2. 3-28 ϕ
3. 2-25 ϕ + 2-20 ϕ
4. 2-28 ϕ + 2-16 ϕ

4. Doubly Reinforced Rectangular Section

4.1 Introduction:

If reinforcement is provided in tension and compression zone of section then such section is called as doubly reinforced. It is provided if section size is restricted (b & d both) and $BM_u > M_{u,lim}$

4.2 Analysis of Doubly Reinforced Section:



For position of N.A.

$$C = T$$

$$C_c + C_s = T$$

$$[0.36\, f_{ck} \cdot x_u \cdot b - A_{sc} \times 0.45 f_{ck}] + f_{sc} \cdot A_{sc} = 0.87 f_y \cdot A_{st}$$

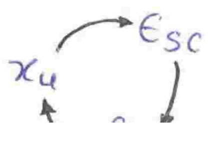
$$0.36\, f_{ck}\, x_u \cdot b + (f_{sc} - 0.45 f_{ck}) \cdot A_{sc} = 0.87\, f_y \cdot A_{st}$$

From above expression x_u cannot be calculated without knowing f_{sc}. Since, f_{sc} depends on ϵ_{sc} so ϵ_{sc} is being calculated from strain diagram as follows.

$$\frac{\epsilon_{sc}}{x_u - d'} = \frac{0.0035}{x_u}$$

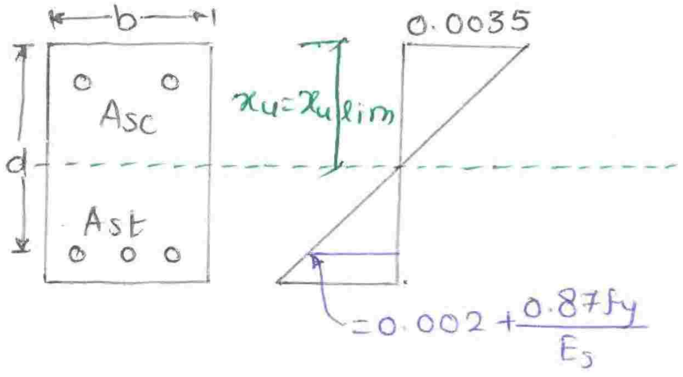
$$\epsilon_{sc} = \left(\frac{x_u - d'}{x_u}\right) \cdot 0.0035$$

In above calculation, x_u depends on f_{sc} and f_{sc} depends on ϵ_{sc}. Since ϵ_{sc} also depends on x_u so this is a cyclic problem which can be solved by trial and error only.



4.3 Position of Neutral Axis:

4.3.1 Balanced Section:



$x_u = x_{u,lim} = 0.53d$ (Fe 250)

$0.48d$ (Fe 415)

$0.46d$ (Fe 500)

4.3.2 Under Reinforced Section:

Step 1: Calculate $x_{u,lim}$ and assume $x_{u_1} = x_{u,lim}$

Step 2: Calculate ϵ_{sc}

$$\epsilon_{sc} = \left(\frac{x_{u_1} - d'}{x_{u_1}}\right) 0.0035$$

Step 3: Take f_{sc} from stress-strain diagram of steel under compression, corresponding to ϵ_{sc}

Stress level	Fe 415		Fe 500	
	Strain	Stress	Strain	Stress
$0.8 f_{yd}$	0.00144	288.7	0.00174	347.8
$0.85 f_{yd}$	0.00163	306.7	0.00195	369.6
$0.90 f_{yd}$	0.00192	324.8	0.00226	391.3
$0.95 f_{yd}$	0.00241	342.8	0.00277	413.0
$0.975 f_{yd}$	0.00276	351.8	0.00312	423.9
$1.0 f_{yd}$	0.00380	360.9	0.00417	434.8

* Note:-

- $f_{yd} \longrightarrow$ Design yield stress $= 0.87 f_y$

- If ϵ_{sc} lies between values of above table then f_{sc} is calculated by linear interpolation.

$$x_1 \longrightarrow y_1$$
$$\epsilon_{sc} \longrightarrow f_{sc}$$
$$x_2 \longrightarrow y_2$$

$$f_{sc} = y_1 + \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(\epsilon_{sc} - x_1)$$

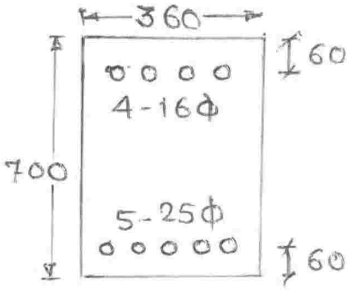
- If ϵ_{sc} is less than 0.00144 and 0.00174 for Fe415 and Fe 500 respectively then f_{sc} is calculated by directly multiplying ϵ_{sc} to E_s

Step 4: Equate net compressive force to net tensile force of section to calculate position of N.A.

$$C = T$$
$$C_c + C_s = T$$
$$0.36 f_{ck}\, x_u \cdot b + (f_{sc} - 0.45 f_{ck}) A_{sc} = 0.87 f_y \cdot A_{st}$$
$$x_u = ??$$

Step 5: If x_u of step 4 is not equal to x_{u_1} then assume $x_{u_2} = x_u$ of step 4 and repeat step 2 to step 4.

Ex. Calculate position of N.A.



M30

Fe415

⇒

I^{st} Trial:

$$x_{u1} = x_{u,lim} = 0.48d$$

$$= 0.48 \times (700 - 60)$$

$$\Rightarrow x_{u1} = 307.2 \text{mm}$$

$$\epsilon_{sc} = \frac{x_{u1} - d'}{x_{u1}} \times 0.0035$$

$$= \frac{307.2 - 60}{307.2} \times 0.0035$$

$$\Rightarrow \epsilon_{sc} = 0.00281$$

$$f_{sc} = y_1 + \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(\epsilon_{sc} - x_1)$$

$$= 351.8 + \left(\frac{360.9 - 351.8}{0.0038 - 0.00276}\right)(0.00281 - 0.00276)$$

$$\Rightarrow f_{sc} = 352.23 \ N/mm^2$$

$$C = T$$

$$0.36 f_{ck} x_u b + (f_{sc} - 0.45 f_{ck}) A_{sc} = 0.87 f_y A_{st}$$

$$0.36 \times 30 \times x_u \times 360 + (352.23 - 0.45 \times 30) \times 4 \times \frac{\pi}{4} \times 16^2$$

$$= 0.87 \times 415 \times 5 \times \frac{\pi}{4} \times 25^2$$

$$\Rightarrow x_u = 157.85 \text{mm} \neq x_{u1} \ (307.2 \text{mm})$$

So, second Trial is required.

IInd Trial:

$\Rightarrow x_{u_2} = x_u = 157.85\,mm$

$$\epsilon_{sc} = \left(\frac{x_{u_2} - d'}{x_{u_2}}\right) \times 0.0035$$

$$= \frac{157.85 - 60}{157.85} \times 0.0035$$

$\Rightarrow \epsilon_{sc} = 0.00217$

$$f_{sc} = y_1 + \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(\epsilon_{sc} - x_1)$$

$$= 324.8 + \left(\frac{342.8 - 324.8}{0.00241 - 0.00192}\right)(0.00217 - 0.00192)$$

$\Rightarrow f_{sc} = 333.97\ N/mm^2$

Now, $C = T$

$$0.36 f_{ck} x_u b + (f_{sc} - 0.45 f_{ck}) A_{sc} = 0.87 f_y A_{st}$$

$$0.36 \times 30 \times x_u \times 360 + (333.97 - 0.45 \times 30) \times 4 \times \frac{\pi}{4} \times 16^2$$

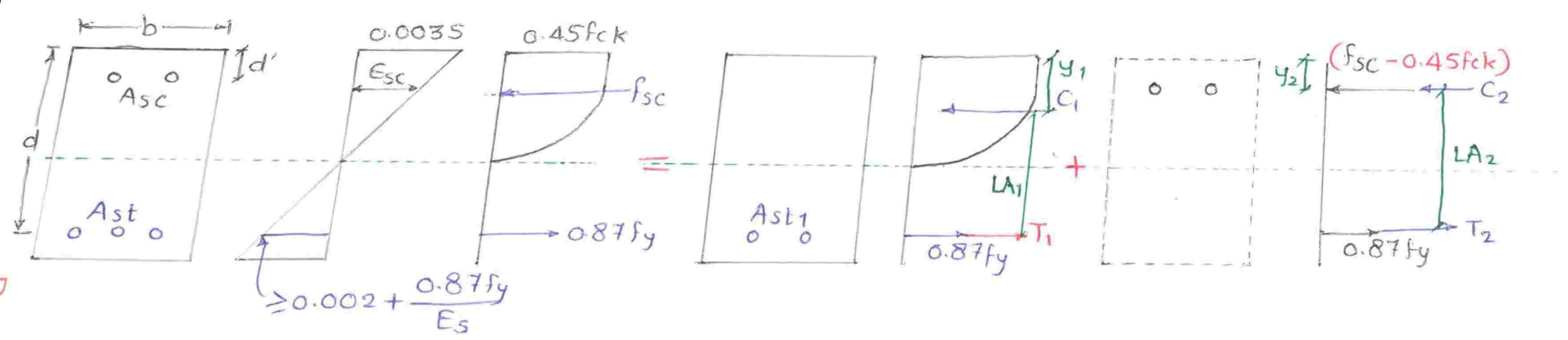
$$= 0.87 \times 415 \times 5 \times \frac{\pi}{4} \times 25^2$$

$\Rightarrow x_u = 161.67\ mm \neq x_{u_2}\ (157.85\,mm)$

So, IIIrd trial is required.

Few more trials are required for relatively closer to exact position of N.A. Assuming $x_u = 160\,mm$ as an approximate position of N.A. to reduce calculation effort.

4.4 Moment of Resistance of Section:



MR from compression Side

$$MR = MR_1 + MR_2$$

$$= C_1 \times LA_1 + C_2 \times LA_2$$

$$MR = 0.36\, f_{ck}\, x_u\, b\,(d - 0.42x_u) + (f_{sc} - 0.45 f_{ck}) A_{sc} \cdot (d - d')$$

MR from Tension Side.

Position of Net compressive force from top fibre $= \bar{y} = \dfrac{C_1 y_1 + C_2 y_2}{C_1 + C_2}$

$$y_1 = 0.42 x_u$$

$$y_2 = d'$$

$$MR = T \times LA$$

$MR = 0.87 f_y\, A_{st}\, (d - \bar{y})$ (Not preferable)

Ex. Calculate MR of section given in previous example.
$x_u = 160$ mm

$\Rightarrow$

$\Rightarrow x_u = 160$ mm

$$\epsilon_{sc} = \left(\frac{x_u - d'}{x_u}\right) \cdot 0.0035$$

$$= \left(\frac{160-60}{160}\right) \times 0.0035$$

$\Rightarrow \epsilon_{sc} = 0.00218$

$$f_{sc} = y_1 + \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(\epsilon_{sc} - x_1)$$

$$= 324.8 + \left(\frac{342.8 - 324.8}{0.00241 - 0.00192}\right)(0.00218 - 0.00192)$$

$\Rightarrow f_{sc} = 334.62 \text{ N/mm}^2$

$$MR = MR_1 + MR_2$$

$$= C_1 \times LA_1 + C_2 \times LA_2$$

$$= 0.36 f_{ck} x_u b (d - 0.42 x_u) + (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d')$$

$$= 0.36 \times 30 \times 160 \times 360 \times (640 - 0.42 \times 160)$$

$$+ (334.62 - 0.45 \times 30) \times 4 \times \frac{\pi}{4} \times 16^2 \times (640 - 60)$$

$\Rightarrow$ M.R. $= 506.1$ kN m

4.5 Design of Doubly Reinforced Rectangular Section:

Doubly reinforced sections are designed if section size is restricted. It means, section size is known and steel is to be designed.

Step 1: Calculate design/factored/ultimate bending moment.

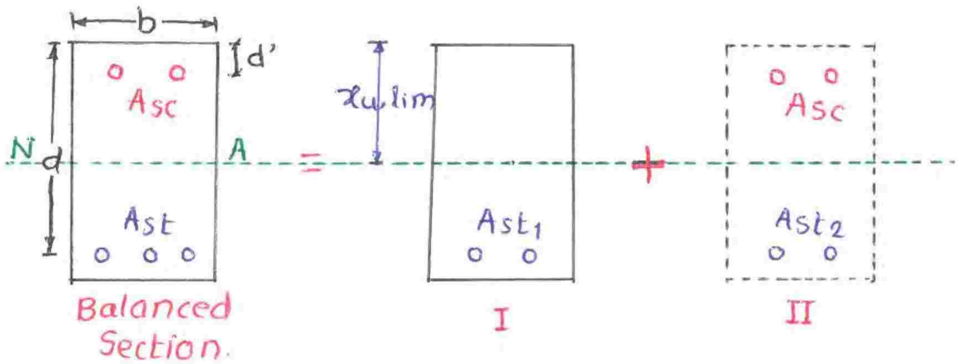
Step 2: Calculate $M_{u,lim}$ of given section.

$$M_{u,lim} = Q b d^2$$

Step 3: If $BM_u \leq M_{u,lim}$ then singly under reinforced section is designed.

$$A_{st} = \frac{0.5 f_{ck} bd}{f_y}\left[1 - \sqrt{1 - \frac{4.6 BM_u}{f_{ck} bd^2}}\right]$$

If $BM_u > M_{u,lim}$ then doubly reinforced balanced section is designed.



Step 4: Calculate A_{st_1}

$$A_{st_1} = A_{st,lim} = 0.414\left(\frac{f_{ck}}{f_y}\right) x_{u,lim} \cdot b$$

Step 5: Calculate Moment resisted by Section II

$$BM_u = MR = MR_1 + MR_2$$

$$MR_2 = BM_u - M_{u,lim} \quad \cdots\cdots \quad (\because MR_1 = M_{u,lim})$$

Step 6: Calculate A_{st_2}

$$MR_2 = T_2 \times LA_2$$

$$BM_u - M_{u,lim} = 0.87 f_y A_{st_2} \cdot (d - d')$$

$$A_{st_2} = ??$$

Step 7: Calculate A_{sc}

$$C_2 = T_2$$

$$(f_{sc} - 0.45 f_{ck}) A_{sc} = 0.87 f_y A_{st_2}$$

$$A_{sc} = ??$$

Step 8: A_{sc} and A_{st} calculated above should be within permissible limits.

$A_{sc,min}$ = No value

$A_{sc,max} = 0.04bD$

$A_{st,min}$ = No need to check

$A_{st,max} = 0.04bD$

Ex. Design a section for factored BM 110 kN·m if size is restricted to 250 x 380 mm. Effective cover 50 mm, M20, Fe415.

⇒

Step 1: ⇒ $\boxed{BM_u = 110 \text{ kN·m}}$

Step 2: $M_{u,lim} = 0.138 f_{ck} bd^2$

$= 0.138 \times 20 \times 250 \times (380-50)^2$

⇒ $\boxed{M_{u,lim} = 75.14 \text{ kNm}}$

Since, $BM_u > M_{u,lim}$ so doubly reinforced balanced section is designed.

Step 3: $A_{st_1} = A_{st,lim} = 0.414 \left(\frac{f_{ck}}{f_y}\right) x_{u,lim} \cdot b$

$= 0.414 \times \left(\frac{20}{415}\right) \times 0.48 \times 330 \times 250$

⇒ $\boxed{A_{st_1} = 790.1 \text{ mm}^2}$

Step 4: $MR_2 = BM_u - M_{u,lim}$

$= 110 - 75.14$

⇒ $\boxed{MR_2 = 34.86 \text{ kN·m}}$

Step 5: $MR_2 = T_2 \times LA_2$

$= 0.87 f_y A_{st_2} \times (d-d')$

$MR_2 = 0.87 \times 415 \times A_{st_2} \times (330-50) = 34.86 \times 10^6$

⇒ $\boxed{A_{st_2} = 344.8 \text{ mm}^2}$

Step 6: $x_{u,lim} = 0.48d = 0.48 \times 330$

$x_{u,lim} = 158.4 \text{ mm}$

$\epsilon_{sc} = \left(\frac{x_{u,lim} - d'}{x_{u,lim}}\right) \times 0.0035 = \left(\frac{158.4 - 50}{158.4}\right) \times 0.0035$

$\epsilon_{sc} = 0.00239$

$$f_{sc} = y_1 + \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(\epsilon_{sc} - x_1)$$

$$= 324.8 + \left(\frac{342.8 - 324.8}{0.00241 - 0.00192}\right)(0.00239 - 0.00192)$$

$f_{sc} = 342.06\ N/mm^2$

Now, $C_2 = T_2$

$(f_{sc} - 0.45 f_{ck})\, A_{sc} = 0.87\, f_y \cdot A_{st_2}$

$(342.06 - 0.45 \times 20) \times A_{sc} = 0.87 \times 415 \times 344.8$

$\Rightarrow \boxed{A_{sc} = 373.77\ mm^2}$

Step 7: $A_{st} = A_{st_1} + A_{st_2} = 790.1 + 344.8$

$\boxed{\begin{array}{l} A_{st} = 1134.9\ mm^2 \\ A_{sc} = 373.77\ mm^2 \end{array}}$

$A_{sc,max} = A_{st,max} = 0.04\, bD = 0.04 \times 250 \times 380$

$= 3800\ mm^2$

$\Rightarrow$ Providing $A_{st} = 2 - 25\phi + 1 - 16\phi$

$A_{sc} = \cancel{2 - 16\phi} \Rightarrow 2 - 20\phi$

Compression steel is increased by more amount than increase in tension steel to make section under-reinforced because $f_{sc} < 0.87 f_y$

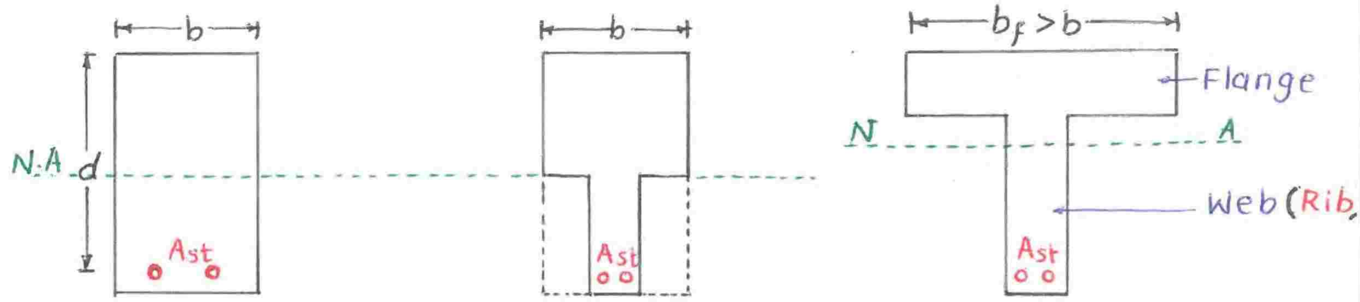
*Note :

- If reduction of concrete area due to presence of steel is not considered then term $0.45 f_{ck} A_{sc}$ is removed from all expressions.
- If table of f_{sc} is not given in examination then assume f_{sc} between $0.75 f_y$ to $0.8 f_y$ for HYSD and $0.87 f_y$ for mild Steel.

5. Flanged Section

5.1 Introduction:

Flanged section is economical and efficient than rectangular section so wherever possible, rectangular section is replaced by flanged section.



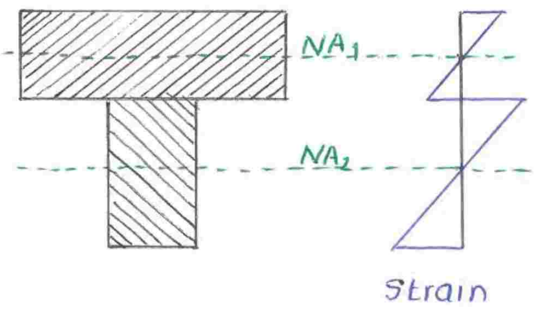
$A_1 > A_2 = A_3$ (economical)

$LA_1 = LA_2 < LA_3$

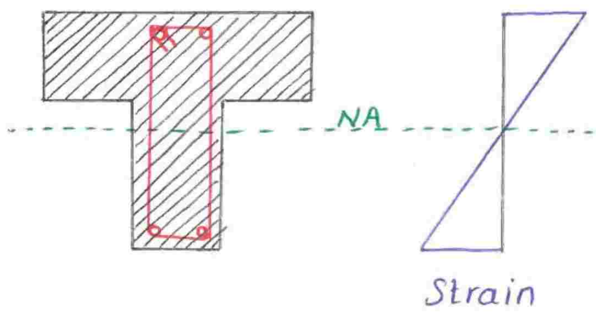
$MR_1 = MR_2 < MR_3$ (efficient)

5.2 Types of Flanged Section:

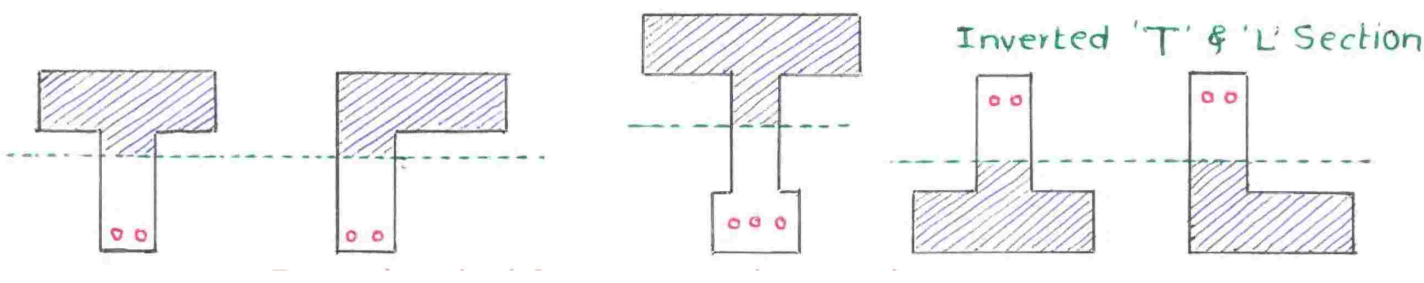
A section is considered as flanged if there is no relative movement between flange and web.

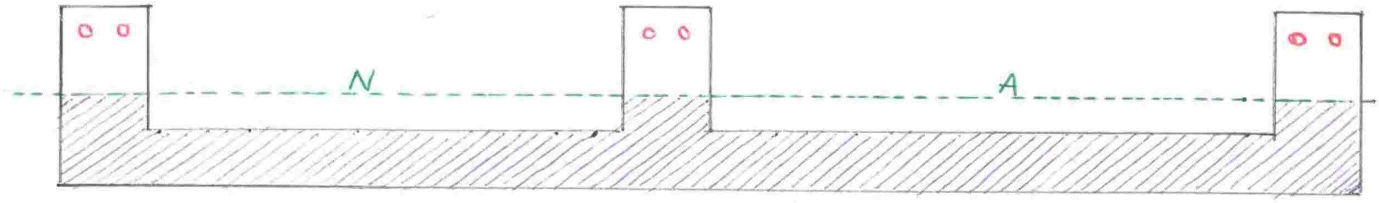


Not a flanged section because of relative movement between flange and Web



Flanged section because of monolithic casting



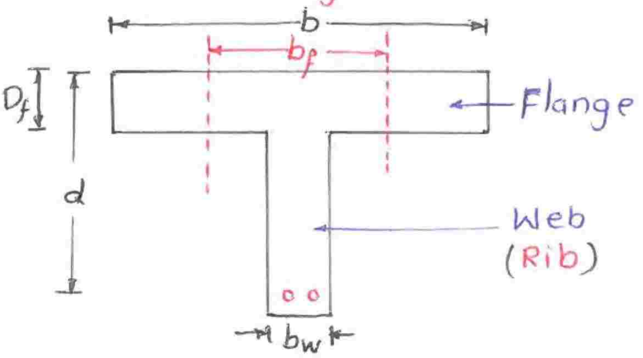


Porch subjected to Hogging B.M.

5.3 Effective flange Width:

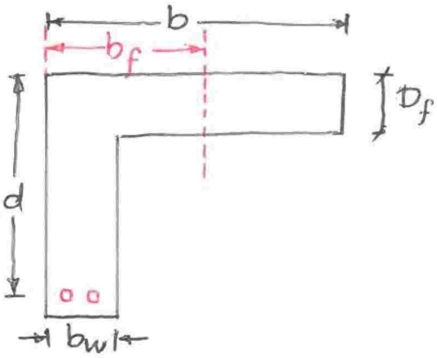
If width of flange is excessive then a part of flange only provides compressive force so effective flange width is calculated.

5.3.1 Isolated Flange Section:



$$b_f = \text{Minimum} \begin{cases} \text{i)}\ \dfrac{l_o}{\dfrac{l_o}{b}+4} + b_w \\ \text{ii)}\ b \end{cases}$$

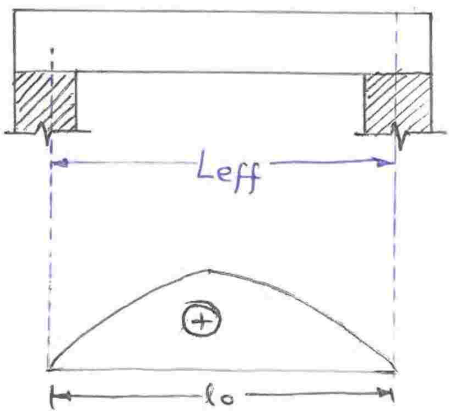
'T' Section



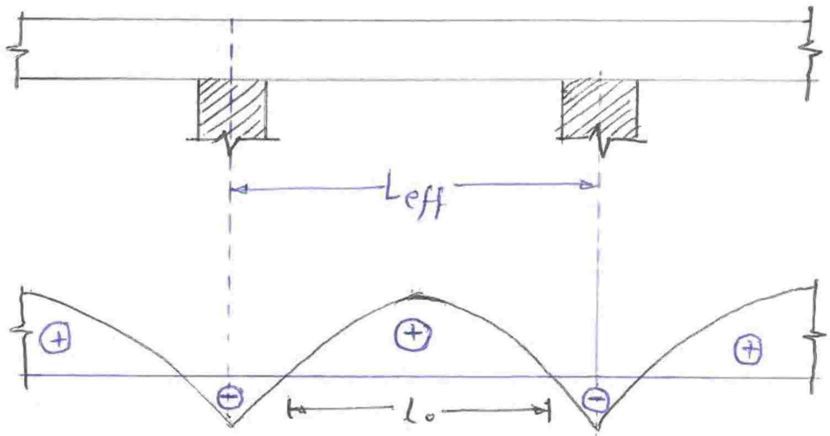
$$b_f = \text{Minimum} \begin{cases} \text{i)}\ \dfrac{0.5\, l_o}{\dfrac{l_o}{b}+4} + b_w \\ \text{ii)}\ b \end{cases}$$

'L' - Section

Where l_o is distance between point of zero moment.

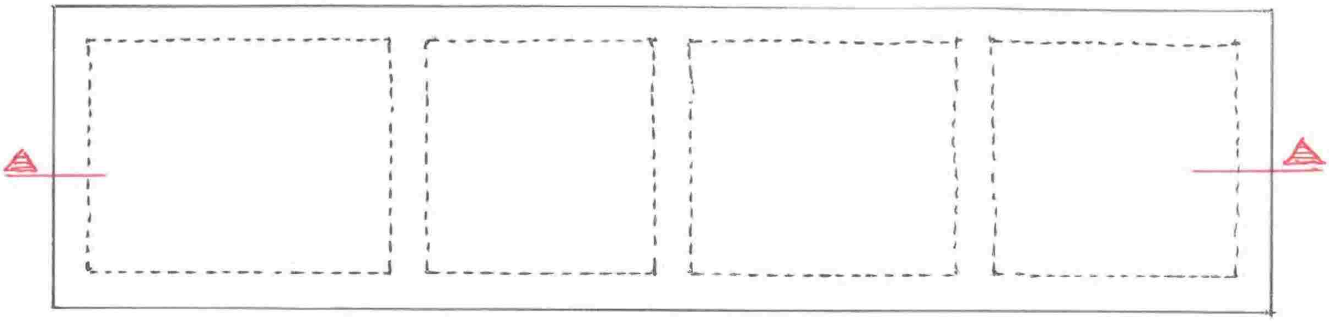


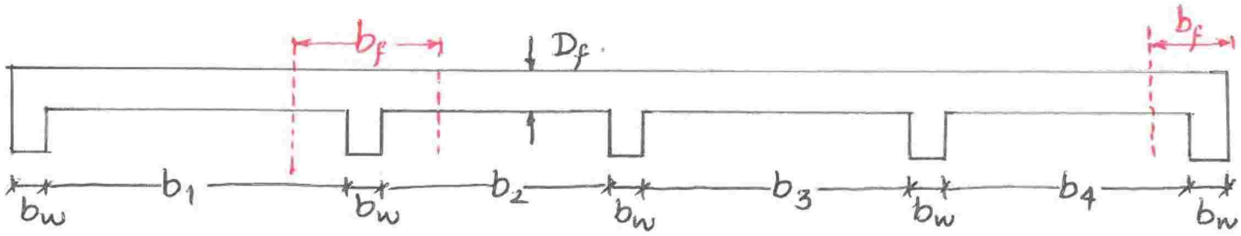
$l_o = L_{eff}$



$l_o = 0.7 L_{eff}$

5.3.2 Beam-Slab Floor System:





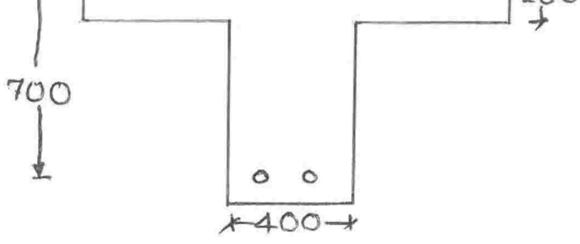
'T' Section:

$$b_f = \text{Minimum}\begin{cases} \text{i)}\ \dfrac{l_o}{6} + b_w + 6D_f \\ \text{ii)}\ \dfrac{b_1}{2} + b_w + \dfrac{b_2}{2} \end{cases}$$

'L' Section:

$$b_f = \text{Minimum}\begin{cases} \text{i)}\ \dfrac{l_o}{12} + b_w + 3D_f \\ \text{ii)}\ \dfrac{b_4}{2} + b_w \end{cases}$$

Ex. Calculate effective width of flange for an isolated T-beam spaning over simply supported effective span 8m. Section details are given below.



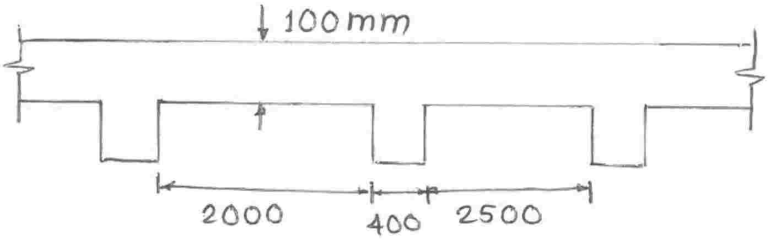
$\Rightarrow$

$$b_f = \text{Minimum}\begin{cases} \text{i)}\ \dfrac{l_o}{\dfrac{l_o}{b} + 4} + b_w = \dfrac{8000}{\dfrac{8000}{2000} + 4} + 400 = 1400\ \text{mm.} \\ \text{ii)}\ b = 2000\text{mm} \end{cases}$$

$$\Rightarrow \boxed{b_f = 1400\text{mm}}$$

Ex. Calculate effective flange width of continuous beam over effective span 30 m. Section details are given below.

⇒



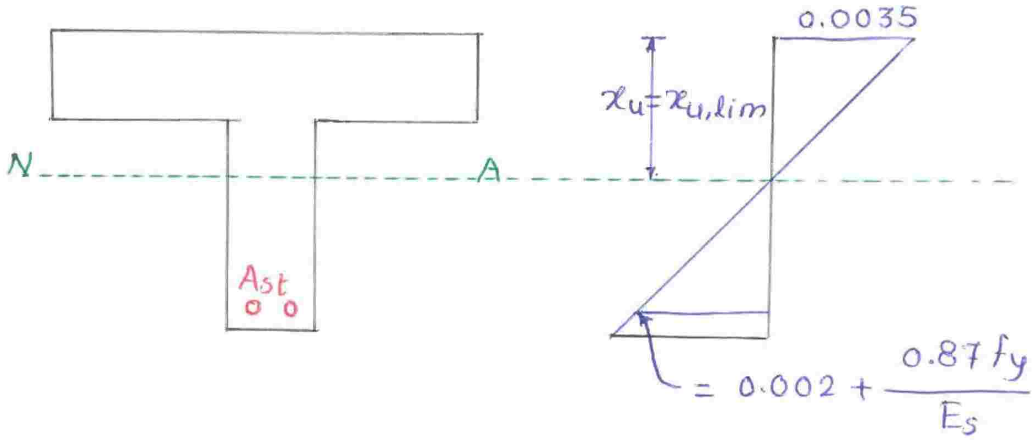
⇒

$$b_f = \text{Minimum}\begin{cases} \dfrac{l_0}{6} + b_w + 6D_f = \dfrac{0.7\times 30\times 10^3}{6} + 400 + 6\times 100 = 4500\text{mm} \\ \dfrac{b_1}{2} + b_w + \dfrac{b_2}{2} = \dfrac{2000}{2} + 400 + \dfrac{2500}{2} = 2650\text{mm.} \end{cases}$$

$$\boxed{b_f = 2650\text{ mm}}$$

5.4 Analysis of Flanged Section:

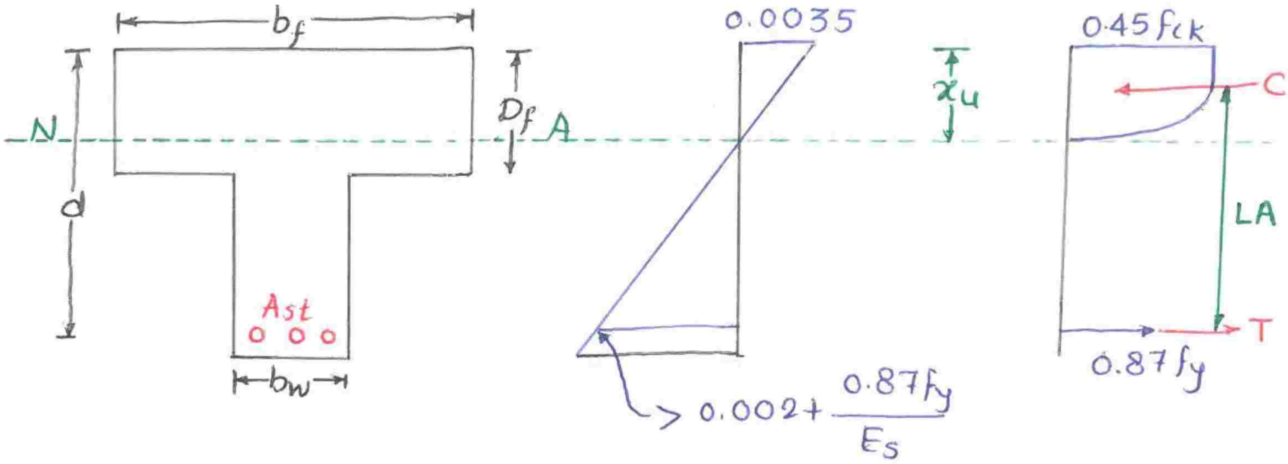
5.4.1 Balanced Section:



$$x_u = x_{u,lim} = \begin{array}{ll} 0.53d & (\text{Fe } 250) \\ 0.48d & (\text{Fe } 415) \\ 0.46d & (\text{Fe } 500) \end{array}$$

5.4.2 Under Reinforced Section:

5.4.2.1 Neutral axis is in Flange: ($x_u \leq D_f$)



For position of NA.

$$C = T$$

$$0.36 f_{ck} \, x_u \cdot b_f = 0.87 f_y \, A_{st}$$

$$\Rightarrow \quad x_u = ?? \quad \leq D_f$$

Now,

$$MR = C \times LA$$

$$MR = 0.36 f_{ck} \, x_u \, b_f \cdot (d - 0.42 x_u)$$

$$MR = T \times LA$$

$$MR = 0.87 f_y \, A_{st} \, (d - 0.42 x_u)$$

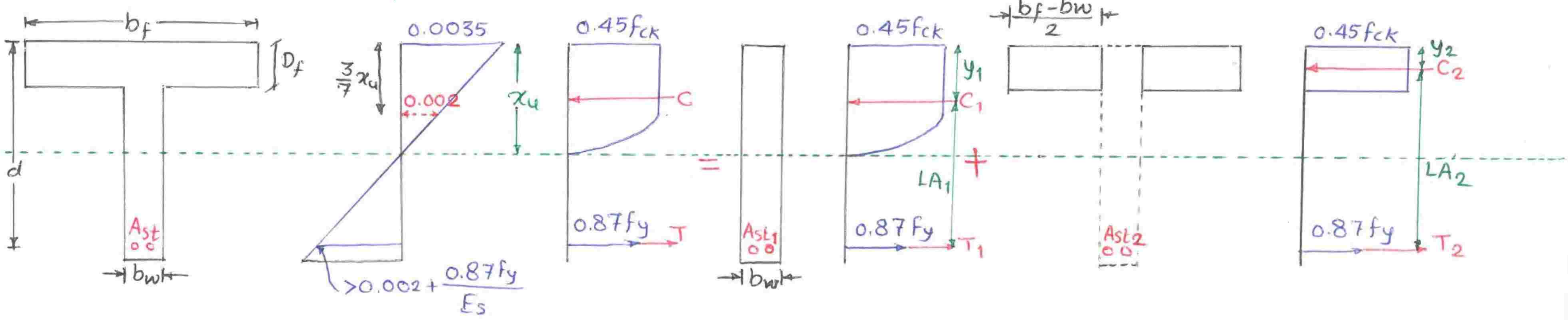
If NA lies in flange then flanged section behaves as rectangular section.

5.4.2.2 Neutral axis is in Web: ($x_u > D_f$)

Case I: $x_u > D_f$ and $D_f \leq \frac{3}{7} x_u$

Case II: $x_u > D_f$ and $D_f \geq \frac{3}{7} x_u$

Case I: $x_u > D_f$ and $D_f \leq \frac{3}{7} x_u$



For position of NA.

$$C = T$$

$$\Rightarrow \quad C_1 + C_2 = T$$

$$\Rightarrow 0.36 f_{ck} x_u \cdot b_w + 0.45 f_{ck} D_f (b_f - b_w) = 0.87 f_y A_s$$

$$x_u = ?? > D_f$$

and $D_f \leq \frac{3}{7} x_u$

* Moment of Resistance (MR)

- MR from compression side:

$$MR = MR_1 + MR_2$$
$$= C_1 \times LA_1 + C_2 \times LA_2$$

$$MR = 0.36 f_{ck} x_u b_w (d - 0.42 x_u) + 0.45 f_{ck} \cdot D_f (b_f - b_w) \cdot (d - \frac{D_f}{2})$$

- MR from Tension side:

Position of Net compressive force from top fibre $= \bar{y} = \frac{C_1 y_1 + C_2 y_2}{C_1 + C_2}$

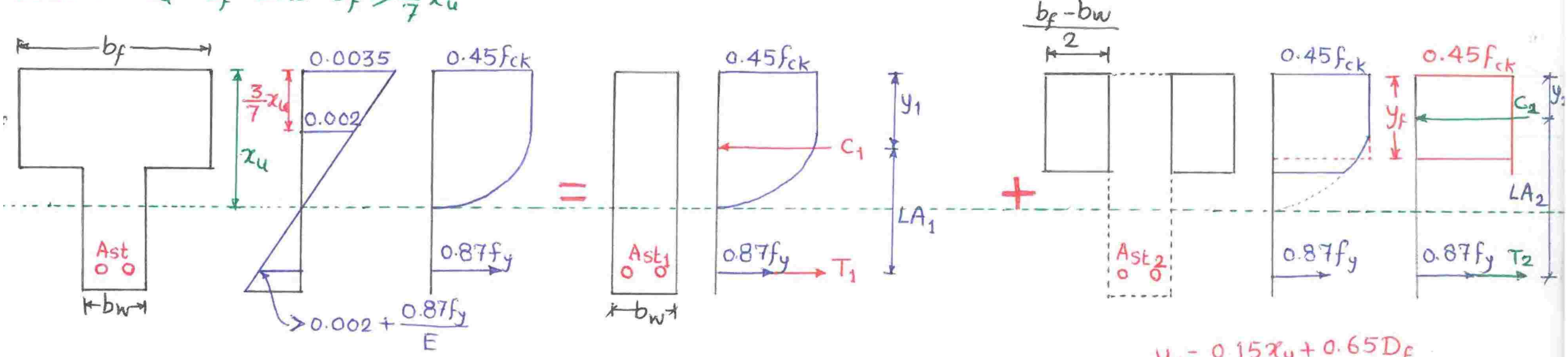
$$y_1 = 0.42 x_u$$
$$y_2 = \frac{D_f}{2}$$

$$MR = T \times LA$$

$$MR = 0.87 f_y \cdot A_{st} \cdot (d - \bar{y}) \quad \text{---- (Not preferable)}$$

5-6

Case II : $x_u > D_f$ and $D_f \geq \frac{3}{7} x_u$



$y_f = 0.15 x_u + 0.65 D_f$

* For Position of NA.

$$C = T$$

$$\Rightarrow \quad C_1 + C_2 = T$$

$$0.36 f_{ck} x_u b_w + 0.45 f_{ck} y_f (b_f - b_w) = 0.87 f_y A_{st}$$

$x_u = ?? > D_f$

and $D_f \geq \frac{3}{7} x_u$

* Moment of Resistance (M.R.)

- MR from compression side.

$$MR = MR_1 + MR_2$$
$$= C_1 \times LA_1 + C_2 \times LA_2$$

$$MR = 0.36 f_{ck} x_u \cdot b_w (d - 0.42 x_u) + 0.45 f_{ck} y_f (b_f - b_w)(d - \frac{y_f}{2})$$

- MR from Tension side:

Position of Net compressive force from top fibre $= \bar{y} = \dfrac{C_1 y_1 + C_2 y_2}{C_1 + C_2}$

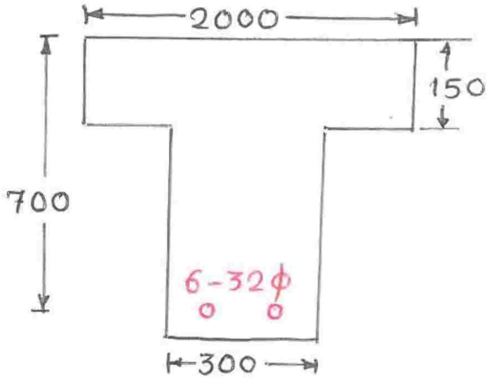
$y_1 = 0.42 x_u$

$y_2 = y_f / 2$

$$MR = T \times LA$$

$MR = 0.87 f_y A_{st} \cdot (d - \bar{y})$ .... (Not preferable)

Ex. Calculate MR of a beam section spaning over simply supported effective span 9m. Section details are given below.



M25

Fe 500

⇒

Step 1:

$$b_f = \text{Minimum}\begin{cases} \text{i)}\ \dfrac{l_o}{\dfrac{l_o}{b}+4} + b_w = \dfrac{9000}{\dfrac{9000}{2000}+4} + 300 \\ \qquad = 1358.82\text{ mm} \\ \text{ii)}\ b = 2000\text{mm} \end{cases}$$

$b_f = 1358.82$ mm

$b_f \approx 1358$ mm

Step 2: Assuming NA is in Flange ($x_u \leqslant D_f$)

$$C = T$$

$$0.36 f_{ck} x_u \cdot b_f = 0.87 f_y \, A_{st}$$

$$0.36 \times 25 \times x_u \times 1358 = 0.87 \times 500 \times 6 \times \frac{\pi}{4} \times 32^2$$

$x_u = 171.75$ mm $>$ D_f (150mm)

So assumption is wrong.

Step 3: Now, assuming NA is in Web ($x_u > D_f$) and $D_f \leqslant \frac{3}{7} x_u$

$$C = T$$

$$0.36 f_{ck} x_u \cdot b_w + 0.45 f_{ck} D_f (b_f - b_w) = 0.87 f_y \cdot A_{st}$$

$$0.36 \times 25 \times x_u \times 300 + 0.45 \times 25 \times 150 \times (1358 - 300) = 0.87 \times 500 \times 6 \times \frac{\pi}{4} \times 32^2$$

$x_u = 116.19$ mm $< D_f$ (150mm)

So assumption is wrong.

Step 4: Now, assuming NA is in web ($x_u > D_f$) and $D_f \geq \frac{3}{7} x_u$.

$$C = T$$

$$C_1 + C_2 = T$$

$$0.36 f_{ck} x_u b_w + 0.45 f_{ck} y_f \cdot (b_f - b_w) = 0.87 f_y \cdot A_{st}$$

$$0.36 \times 25 \times x_u \times 300 + 0.45 \times 25 \times (0.15 x_u + 0.65 \times 150) \times (1358 - 300)$$
$$= 0.87 \times 500 \times 6 \times \frac{\pi}{4} \times 32^2$$

$\boxed{x_u = 209.25 mm} > D_f$ (150 mm)

$\frac{3}{7} x_u = 89.68 mm < D_f$ (150 mm).

$y_f = 0.15 x_u + 0.65 D_f = 0.15 \times 209.25 + 0.65 \times 150$

$y_f = 128.88 mm$

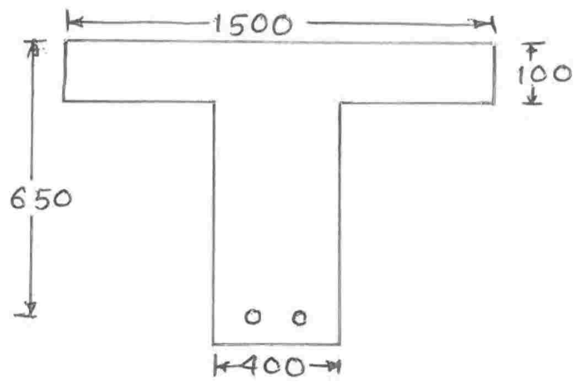
Step 5: $MR = MR_1 + MR_2$

$= C_1 \times LA_1 + C_2 \times LA_2$

$$MR = 0.36 f_{ck} x_u b_w (d - 0.42 x_u) + 0.45 f_{ck} y_f (b_f - b_w)(d - \frac{y_f}{2})$$
$$= 0.36 \times 25 \times 209.25 \times 300 \times (700 - 0.42 \times 209.25)$$
$$+ 0.45 \times 25 \times 128.88 \times (1358 - 300) \times (700 - \frac{128.88}{2})$$

$\boxed{MR = 1320.77 kN \cdot m}$

Ex Design a section given below for ultimate BM 650 kN·m. This section is used in a simply supported beam of effective span of 7 m



M20

Fe 415

⇒

$$b_f = \text{Minimum}\begin{cases} \text{i)}\ \dfrac{l_o}{\frac{l_o}{b}+4} + b_w = \dfrac{7000}{\frac{7000}{1500}+4} + 400 = 1207.69 \\ \text{ii)}\ b = 1500\,\text{mm} \end{cases}$$

$b_f = 1207.69$ mm

$\boxed{b_f \approx 1207\ \text{mm}}$

Step 1: ⇒ $\boxed{BM_u = 650\ \text{kN·m}}$

Step 2: $x_{u,lim} = 0.48d$

$= 0.48 \times 650$

⇒ $\boxed{x_{u,lim} = 312\ \text{mm}}$

Since $x_{u,lim} > D_f$ & $D_f < \frac{3}{7} x_{u,lim}$

$M_{u,lim}$ = Case I of 5.4.2.2

$= C_1 \times LA_1 + C_2 \times LA_2$

$$M_{u,lim} = 0.36 f_{ck}\, x_{u,lim}\, b_w (d - 0.42\, x_{u,lim}) + 0.45 f_{ck} \cdot D_f\,(b_f - b_w) \times \left(d - \frac{D_f}{2}\right)$$

$$= 0.36 \times 20 \times 312 \times 400 \times (650 - 0.42 \times 312) + 0.45 \times 20 \times 100 \times (1207 - 400) \times \left(650 - \frac{100}{2}\right)$$

⇒ $\boxed{M_{u,lim} = 902.1\ \text{kN-m}}$

Step 3: Since $BM_u < M_{u,lim}$ so singly under reinforced section is designed.

Step 4: Assuming $x_u = D_f = 100\ mm$

$$MR_1 = C \times L.A \qquad (5.4.2.1)$$

$$MR_1 = 0.36\, f_{ck}\, x_u \cdot b_f\, (d - 0.42 x_u)$$

$$= 0.36 \times 20 \times 100 \times 1207 \times (650 - 0.42 \times 100)$$

$$\Rightarrow \boxed{MR_1 = 528.37\ kN{\cdot}m}$$

Step 5: Since $MR_1 < BM_u < M_{u,lim}$ so assuming $x_u = \frac{7}{3} D_f$

$$MR_2 = C_1 \times LA_1 + C_2 \times LA_2 \cdots \text{(case I of 5.4.2.2)}$$

$$MR_2 = 0.36 f_{ck} \cdot x_u \cdot b_w (d - 0.42 x_u) + 0.45 f_{ck} \cdot D_f\, (b_f - b_w)\,(d - \frac{D_f}{2})$$

$$= 0.36 \times 20 \times \frac{7}{3} \times 100 \times 400 \times (650 - 0.42 \times \frac{7}{3} \times 100)$$

$$+ 0.45 \times 20 \times 100 \times (1207 - 400) \times (650 - \frac{100}{2})$$

$$\Rightarrow \boxed{MR_2 = 806.7\ kN{\cdot}m.}$$

Step 6: Since $MR_1 < BM_u < MR_2$

For exact position of NA

$BM_u = MR$ (case II of 5.4.2.2)

$BM_u = C_1 \times LA_1 + C_2 \times LA_2$

$$BM_u = 0.36 f_{ck} \cdot x_u \cdot b_w (d - 0.42 x_u) + 0.45 f_{ck} \cdot y_f\, (b_f - b_w)\,(d - \frac{y_f}{2})$$

$$650 \times 10^6 = 0.36 \times 20 \times x_u \times 400 \times (650 - 0.42 \times x_u)$$

$$+ 0.45 \times 20 \times \left[0.15\, x_u + (0.65 \times 100)\right] \times (1207 - 400)$$

$$\times \left[650 - \left(\frac{0.15\, x_u + 0.65 \times 100}{2}\right)\right]$$

$x_u = 155.26\ mm > D_f\ (100mm)$

and $\frac{3}{7} x_u = 66.54\ mm < D_f\ (100mm)$

For Ast : $C = T$

$$C_1 + C_2 = T$$

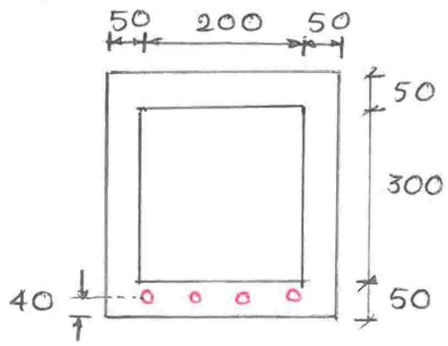
$$0.36 f_{ck}\, x_u \cdot b_w + 0.45 f_{ck}\, y_f \cdot (b_f - b_w) = 0.87 f_y \cdot A_{st}$$

$$0.36 \times 20 \times 155.26 \times 400 + 0.45 \times 20 \times [(0.15 \times 155.26) + (0.65 \times 100)] \times (1207 - 400)$$

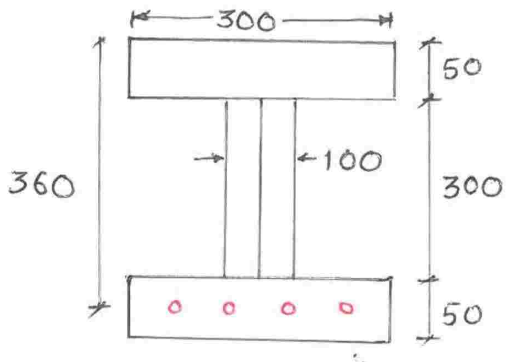
$$= 0.87 \times 415 \times A_{st}.$$

$\Rightarrow \boxed{A_{st} = 3014.5\ mm^2.}$

Ex. Design the section given below for factored B.M. 40 kN-m
M25, Fe 415.



$\Rightarrow$ Equivalent Section:



$b_f = 300mm$
$D_f = 50mm$
$d = 360mm$
$b_w = 100mm$

Step 1: $\Rightarrow \boxed{BM_u = 40\ kN \cdot m}$

Step 2: $x_{u,lim} = 0.48d$

$= 0.48 \times 360$

$\Rightarrow \boxed{x_{u,lim} = 172.8\ mm}$

Since $x_{u,lim} > D_f$ and $D_f < \frac{3}{7} x_{u,lim}$

$M_{u,lim} = C_1 \times LA_1 + C_2 \times LA_2$ ...... (Case I of 5.4.2.2)

$$M_{u,lim} = 0.36 f_{ck} x_{u,lim} \cdot b_w \cdot (d - 0.42 x_{u,lim}) + 0.45 f_{ck} D_f \cdot (b_f - b_w) \cdot (d - \frac{D_f}{2})$$

$$= 0.36 \times 25 \times 172.8 \times 100 \times (360 - 0.42 \times 172.8) + 0.45 \times 25 \times 50 \times (300 - 100) \times (360 - \frac{50}{2})$$

$\Rightarrow \boxed{M_{u,lim} = 82.39 \text{ kN·m}}$

Step 3. Since $BM_u < M_{u,lim}$ singly under reinforced section is designed.

Step 4: Assuming $x_u = D_f = 50mm$

$MR_1 = C \times LA$ .... (5.4.2.1)

$$MR_1 = 0.36 f_{ck} x_u \cdot b_f \cdot (d - 0.42 x_u)$$

$$= 0.36 \times 25 \times 50 \times 300 \times (360 - 0.42 \times 50)$$

$\Rightarrow \boxed{MR_1 = 45.76 \text{ kN·m}}$

Step 5: Since $BM_u < MR_1$ then NA is in flange and section behaves as rectangular section

$$A_{st} = \frac{0.5 f_{ck} b_f d}{f_y} \left[1 - \sqrt{1 - \frac{4.6 BM_u}{f_{ck} b_f d^2}} \right]$$

$$= \frac{0.5 \times 25 \times 300 \times 360}{415} \times \left[1 - \sqrt{1 - \frac{4.6 \times 40 \times 10^6}{25 \times 300 \times 360^2}} \right]$$

$\Rightarrow \boxed{A_{st} = 324.04 mm^2}$

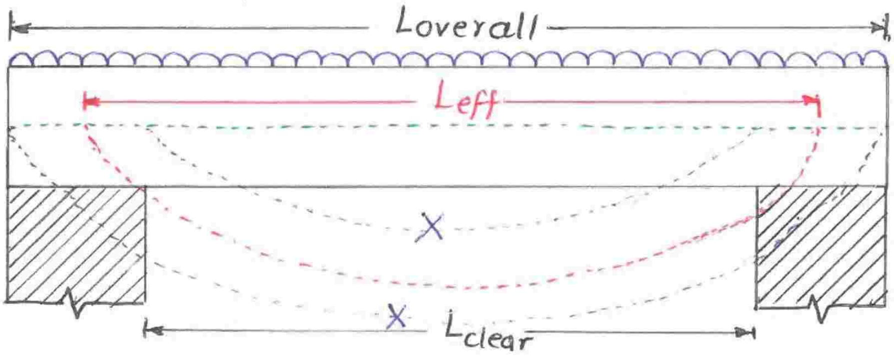
6. Design of Beam

6.1 Introduction:

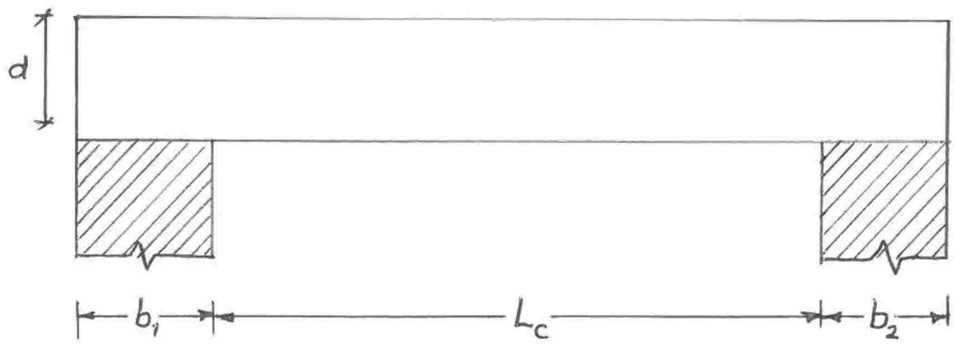
In previous chapters, analysis and design of section were covered. In this chapter, design of complete beam is going to be discussed.

6.2 Effective Span:

Portion of beam that effectively participates in bending is called effective span.



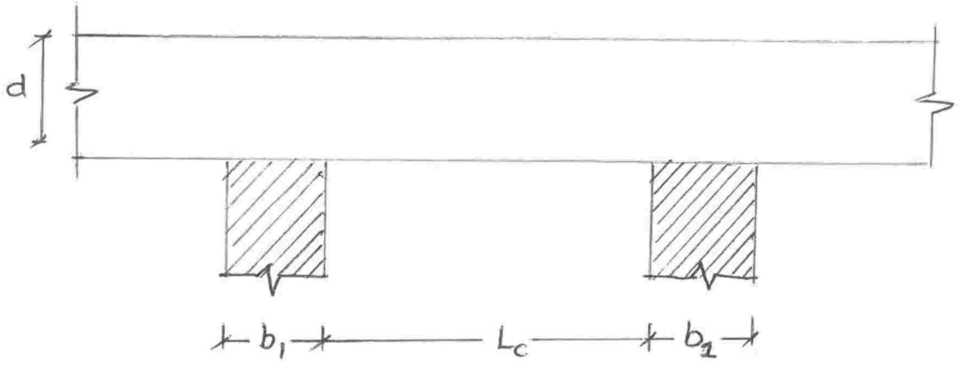
6.2.1 Simply Supported Beam/Slab:



$$L_{eff} = \text{Minimum} \begin{cases} \text{i)} \; L_c + d \\ \text{ii)} \; \dfrac{b_1}{2} + L_c + \dfrac{b_2}{2} \end{cases}$$

6.2.2 Continuous Beam:

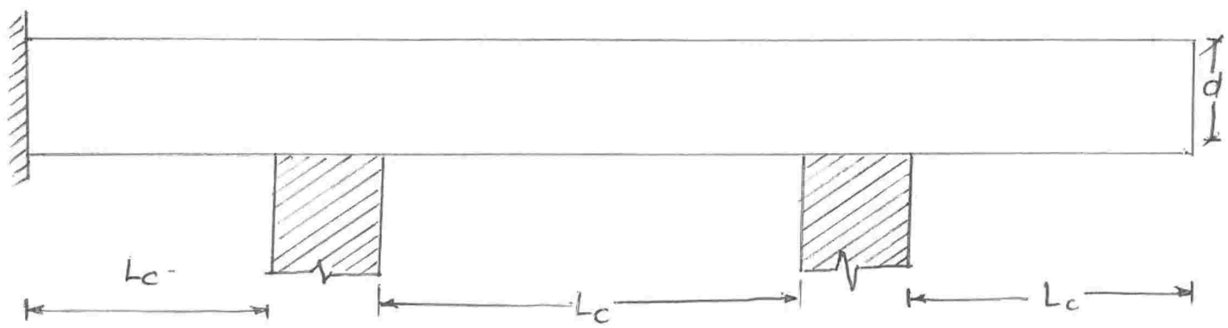
Case I: If support width is less than $\frac{L_c}{12}$ then Leff is same as 6.2.1



b_1 & $b_2 < \frac{L_c}{12}$

$$L_{eff} = \text{Minimum} \begin{cases} \text{i)}\ L_c + d \\ \text{ii)}\ \frac{b_1}{2} + L_c + \frac{b_2}{2} \end{cases}$$

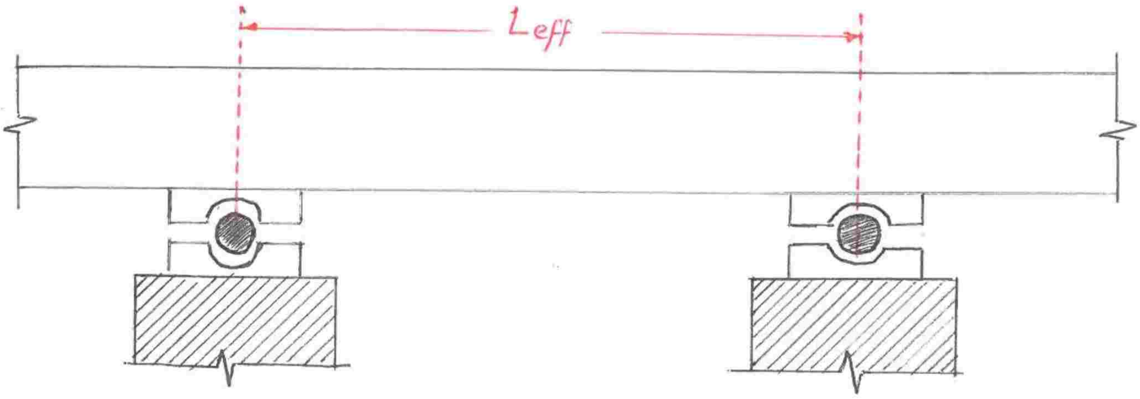
Case II: If support width is more than $\frac{L_c}{12}$ or 600 mm whichever is smaller then L_{eff} is calculated as follows.



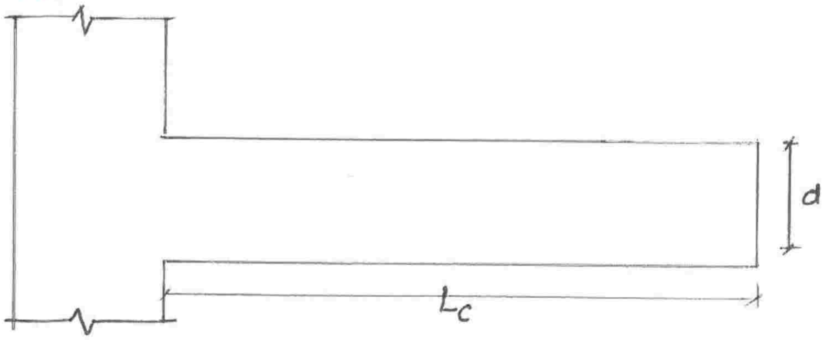
$L_{eff} = L_c$ $\quad$ $L_{eff} = L_c$

$$L_{eff} = \text{Minimum} \begin{cases} \text{i)}\ L_c + \frac{d}{2} \\ \text{ii)}\ L_c + \frac{b}{2} \end{cases}$$

Case III: If beam is continuous over Roller-rocker bearing then L_{eff} is c/c distance between bearings.



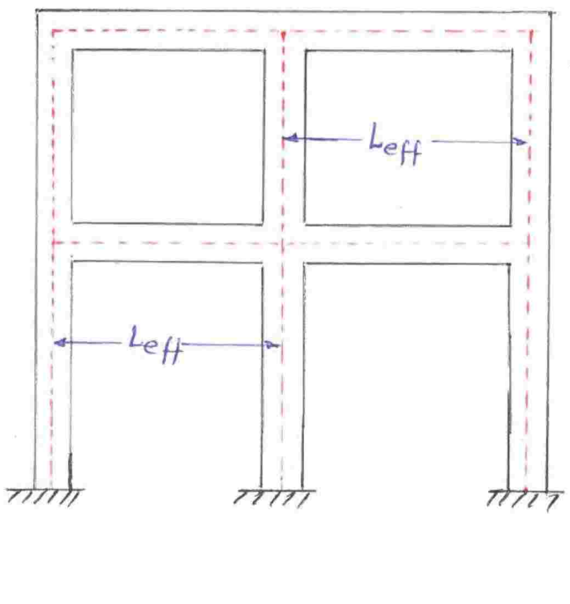
6.2.3 Cantilever:



$$L_{eff} = L_c + \frac{d}{2}$$

6.2.4 Rigid Frames:

L_{eff} is c/c distance between columns.



6.3 Longitudinal Reinforcement:

6.3.1 Longitudinal Tension Reinforcement:

$$\frac{A_{st,min}}{bd} > \frac{0.85}{f_y} \quad \cdots\cdots \left(\text{To prevent sudden failure and for ductility}\right)$$

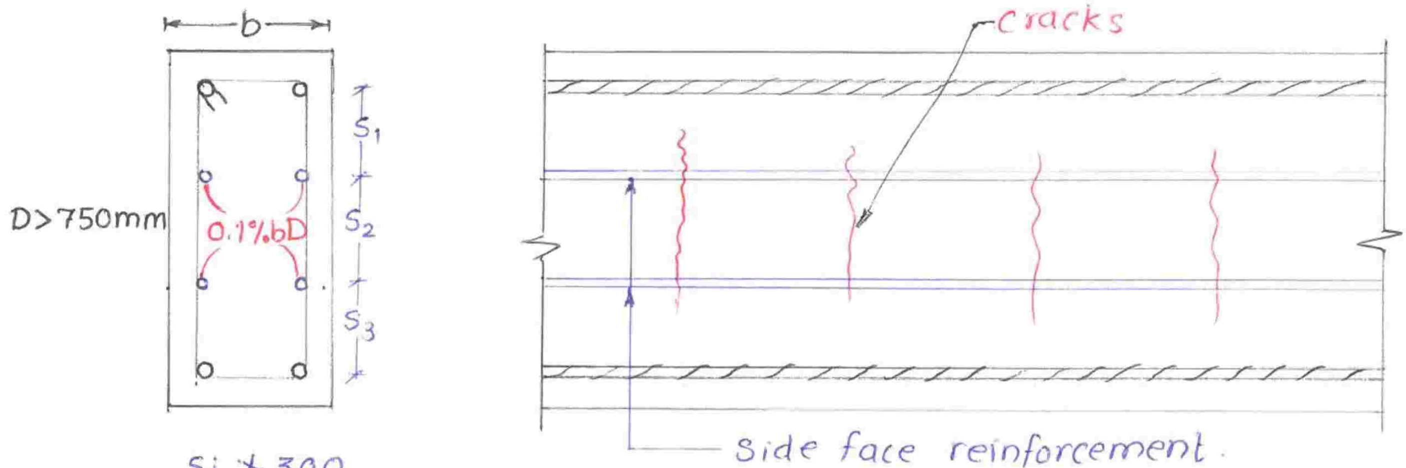
$$A_{st,max} = 0.04\,bD \quad \cdots\cdots (\text{Problem in compaction})$$

6.3.2 Longitudinal Compression Reinforcement:

$A_{sc,min}$ = No value, but atleast 2-bars must be provided in compression zone for ductility and to hold stirrups.

$$A_{sc,max} = 0.04\,bD \quad \cdots\cdots (\text{Problem in compaction})$$

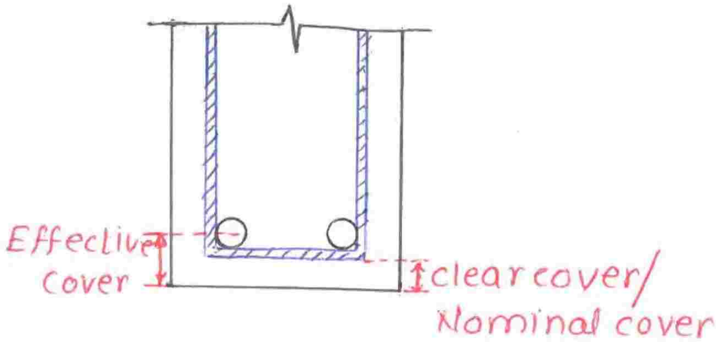
6.3.3 Side Face Reinforcement:

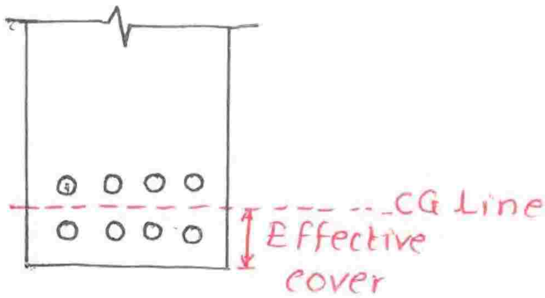


$S_i \ngtr 300$

For $D > 750$mm, 0.1% of gross web area is provided as side face reinforcement, equally distributed on both faces with spacing not more than 300mm. It is provided to take care of shrinkage of concrete along longitudinal direction (vertical cracks) and lateral buckling.

6.4 Minimum Nominal Cover:





- Minimum nominal cover is governed by following criteria.
  1. Type of Structural Member (Beam, slab, footing, etc.)
  2. Exposure condition (Mild, moderate, etc)
  3. Fire resistance in terms of hours (30 min to 4 hrs)

	Slab	Beam	Column	Footing
IS 456	20	20	40	50
SP 34	20	25	40	50

or dia. of bar whichever is greater.

6.5 Maximum Permissible Crack Width in RCC Structure:

Exposure Condition	Maximum Permissible crack width
• Mild (Cracks are not - harmful)	0.3mm
• Moderate & Severe (Cracks are harmful)	0.2mm
• Very Severe & Extreme	0.1mm

6.6 Horizontal Spacing of Reinforcement:

6.6.1 Minimum Horizontal Clear Spacing:

Maximum of following:

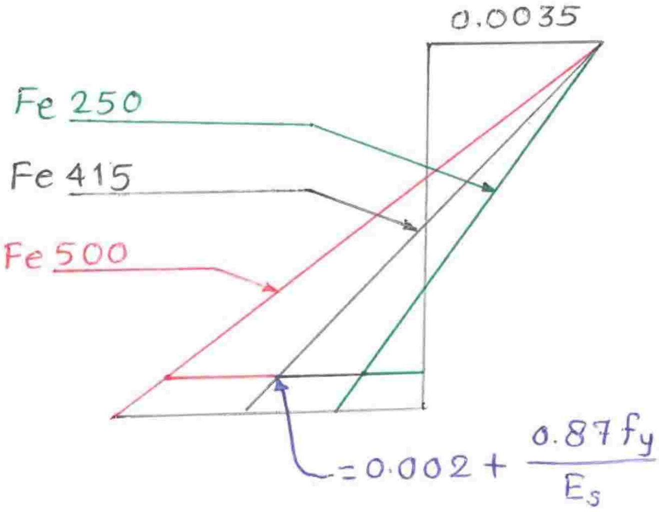
1) Dia. of bar if bars are of equal dia.
2) Dia. of larger dia. bar if bars are of unequal dia.
3) Nominal size of coarse aggregate + 5mm.

6.6.2. Maximum Horizontal c/c spacing:

Fe 250 → 300mm

Fe 415 → 180mm

Fe 500 → 150mm.

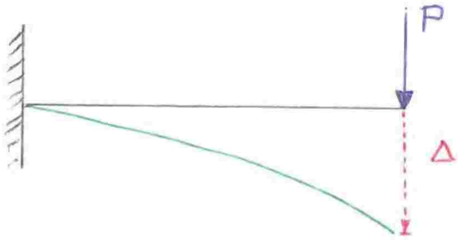


6.10 Total Deflection of Beam:

$$\Delta_{Total} = \underbrace{\Delta_{Loading} + \Delta_{Temp.}}_{\text{Short term}} + \underbrace{\Delta_{Shrinkage} + \Delta_{Creep}}_{\text{Long term}}$$

Short term modulus of Elasticity is used

Long term Modulus of Elasticity is used.



$$\Delta = \frac{PL^3}{3EI}$$

6.11 Deflection Criteria:

6.11.1 Deflection Limits:

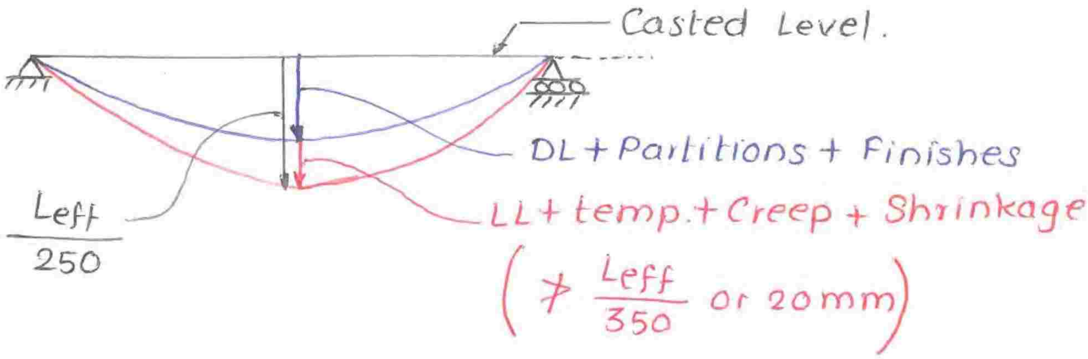
1) The final deflection due to all loads including the effect of temperature, creep and shrinkage and measured from the cast level of the supports of floors, roofs and other horizontal members should not normally exceed

$\frac{L_{eff}}{250}$ (Dead Load + Live Load + Creep + Temp. + Shrinkage)

This limitation is based on crack limitation with which code is very much concerned and to avoid pscycological fear of occupants or affect the appearance of structure

2) The deflection including the effect of temperature, creep and shrinkage occuring after erection of partitions and the application of finishes, should not normally exceed

$\frac{L_{eff}}{350}$ or 20mm whichever is less.



6.112 Deflection Control:

Exact calculation of deflection and keeping it within permissible limits need lot of calculation. IS456 provides a simplified approach to keep deflection of beam within permissible limits.

If $\frac{L_{eff}}{d}$ ratio satisfies following conditions then beam is safe in deflection.

Support condition	Values.
Cantilever	7
Simply Supported	20
Continuous	26

$$\frac{L_{eff}}{d} < k_1\, k_2\, k_3\, k_4 \text{ (value)}$$

where, k_i = Modification factor.

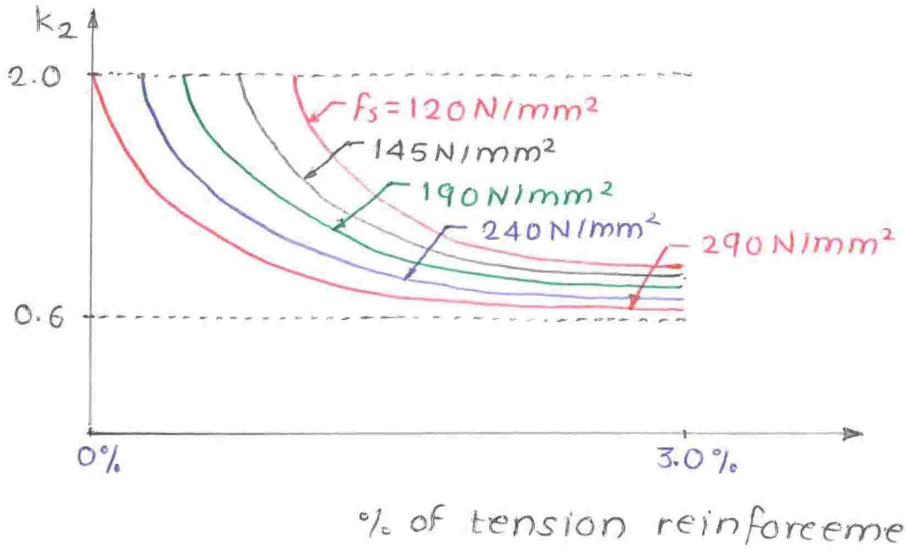
$k_1 \Rightarrow$ Depends on Span

$= 1$ (upto 10m)

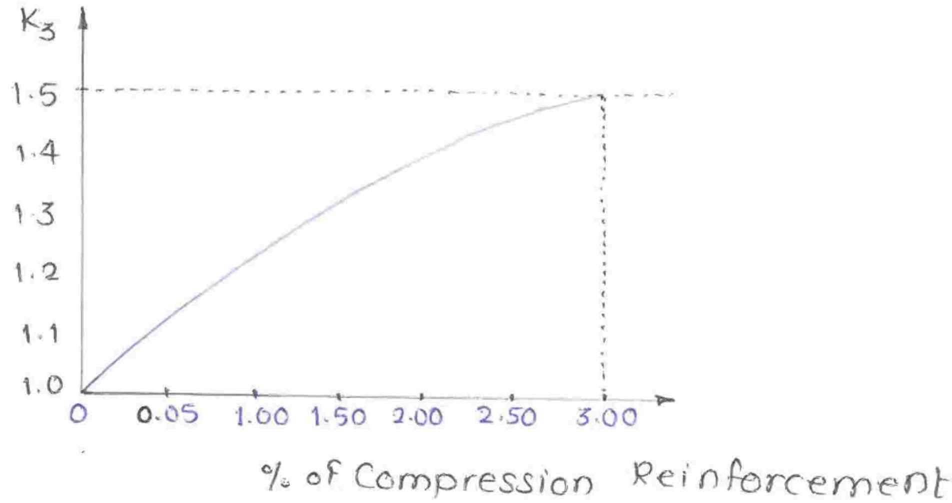
$= \frac{10}{\text{Span(m)}}$ (beyond 10m, NA for cantilever)

$k_2 \Rightarrow$ Depends on % of Tension Reinforcement.

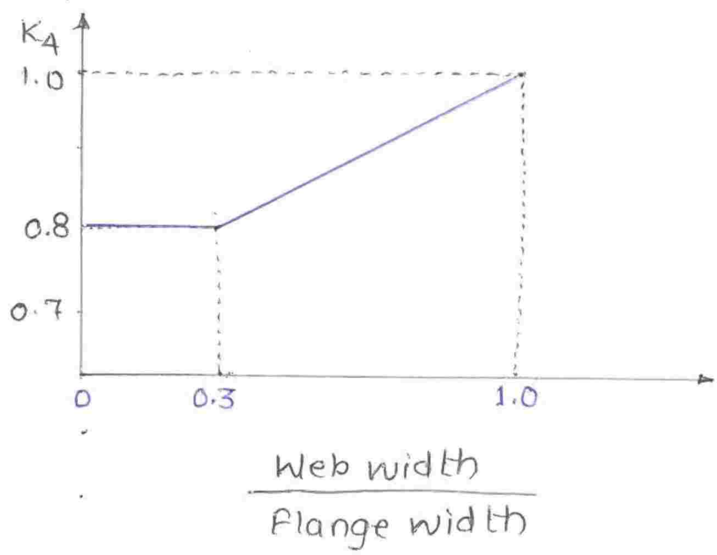
f_s = Stress level of steel $= 0.58 f_y \frac{A_{st,req}}{A_{st,provided}}$



K3 ⇒ Depends on % of Compression Reinforcement:



K4 ⇒ Depends on Ratio of Web width to Flange Width:



*Note:

$$\frac{L_{eff}}{d} < K_1 K_2 K_3 K_4 \text{ (value)}$$

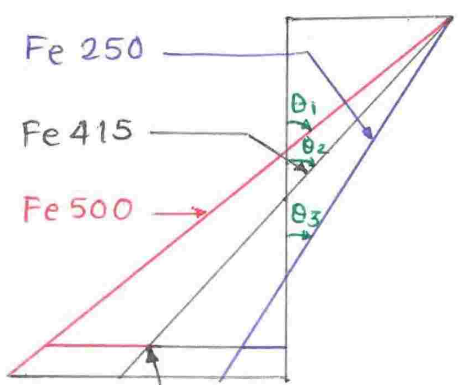
$$d > \frac{L_{eff}}{K_1 K_2 K_3 K_4 \text{ (value)}}$$

- Since K_4 is higher for rectangular section so depth requirement to satisfy deflection criteria is less for rectangular section than flanged section. It means flange section produces more deflection.

-

- As % of compression reinforcement increases, K_3 increases. It means depth requirement to satisfy deflection criteria decreases. In other words higher % of compression reinforcement produces less deflection.

- Higher grade of steel produces more deflection.



$$= 0.002 + \frac{0.87 f_y}{E_s}$$

$\theta_1 > \theta_2 > \theta_3$

$\Delta_{500} > \Delta_{415} > \Delta_{250}$

6.12 Design of Simply Supported Singly Reinforced Beam of Rectangular Section:

Step 1: Assume suitable value of $\frac{b}{d}$ ratio.

(Lateral Buckling) $0.3 < \frac{b}{d} < 0.7$ (uneconomical)

$\frac{b}{d} = 0.5$ (for exam)

Step 2: Assume suitable value of d based on following criteria

- Thumb rule:

$$\frac{L_{eff}}{15} < d < \frac{L_{eff}}{10}$$

- Deflection criteria:

$$\frac{L_{eff}}{d} < k_1 k_2 k_3 k_4 \text{ (value)}$$

Where, $k_1 \rightarrow$ depends on span

$k_2 = 1$ (because % of tension reinforcement is not known)

$k_3 = 1$ (Singly reinforced)

$k_4 = 1$ (Rectangular)

Select suitable value of d based on above two criteria and calculate b and D accordingly.

* Note:

Since L_{eff} is not known so L_{clear} or c/c distance between supports can be used at the place of L_{eff} in above calculation for priliminary design.

Step 3: Calculate effective span.

Step 4: Calculate DL and design B.M.

Step 5: Calculate 'd' required for balanced section

$$BMu = M_{u,lim}$$

$$BMu = Q\,bd^2$$

$$\Rightarrow d = ??$$

'd' calculated here should be less than assumed in step 2. otherwise select 'd' suitably higher than calculated here and repeat Step 3, Step 4 and Step 5.

Step 6: Since section size provided in Step 2 is larger than required in step 5 so provided section is under reinforced.

$$A_{st} = \frac{0.5 f_{ck} bd}{f_y}\left[1 - \sqrt{1 - \frac{4.6\,BMu}{f_{ck}\,bd^2}}\right]$$

Step 7: Ast calculated above should be within permissible limits

Step 8: Provide side face reinforcement if required.

Step 9: Apply check for deflection.

Ex. Design a simply supported singly reinforced beam of rectangular section spanning over clear span 11m and resting on 400mm thick wall on one side and 500mm on another side. It is subjected to superimposed LL of 30kN/m. M30, Fe500, Severe exposure.

⇒

Step 1: $\frac{b}{d} = 0.5$ (Assume)

Step 2: Effective depth

- Thumb rule.

$$\frac{L_{eff}}{15} < d < \frac{L_{eff}}{10}$$

$$\frac{11.45 \times 10^3}{15} < d < \frac{11.45 \times 10^3}{10}$$

$$\Rightarrow 763.33 < d < 1145 \text{mm}.$$

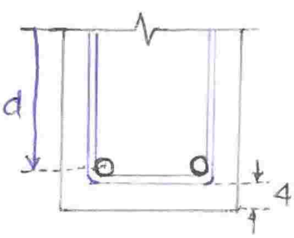
- Deflection Criteria:

$$\frac{L_{eff}}{d} < K_1 \; K_2 \; K_3 \; K_4 \text{ (value)}$$

$$\frac{11.45 \times 10^3}{d} < \left(\frac{10}{11.45}\right) (1)(1)(1)(20)$$

$$\Rightarrow d > 655.51 \text{mm}.$$

⇒ $\boxed{\text{Providing } d = 800\text{mm}, \; b = 0.5d = 400\text{mm}}$



D = d + allowance for reinforcement + Clear cover

= 800 + 30 (assumed) + 45 (severe)

⇒ $\boxed{D = 875\text{mm}}$

Step3: Effective span

$$L_{eff} = \text{Minimum} \begin{cases} \text{i)}\ L_c + d = 11 + 0.8 = 11.8\,m \\ \text{ii)}\ \frac{b_1}{2} + L_c + \frac{b_2}{2} = \frac{0.4}{2} + 11 + \frac{0.5}{2} = 11.45\,m \end{cases}$$

$$\Rightarrow \boxed{L_{eff} = 11.45\,m}$$

Step4: Loading - DL, LL and Factored B.M.

$DL = 0.4 \times 0.875 \times 1 \times 25 = 8.75\ kN/m$

$LL = 30\ kN/m$

Total factored load $= W_u = 1.5 \times (8.75 + 30) = 58.125\ kN/m$

$$BM_{u(\text{mid-span})} = \frac{W_u L^2}{8}$$

$$= \frac{58.125 \times (11.45)^2}{8}$$

$$\boxed{BM_u = 952.54\ kN\cdot m}$$

Step5: 'd' required for balanced section

$BM_u = M_{u,lim}$

$BM_u = 0.133\, f_{ck}\, b d^2$

$952.54 \times 10^6 = 0.133 \times 30 \times 400 \times d^2$

$$\Rightarrow \boxed{d = 772.54\,mm < 800\,mm \Rightarrow OK.}$$

Step 6: A_{st} Required.

$$A_{st} = \frac{0.5 f_{ck} b d}{f_y}\left[1 - \sqrt{1 - \frac{4.6\, BM_u}{f_{ck} b d^2}}\right]$$

$$= \frac{0.5 \times 30 \times 400 \times 800}{500}\left[1 - \sqrt{1 - \frac{4.6 \times 952.54 \times 10^6}{30 \times 400 \times 800^2}}\right]$$

$A_{st} = 3308.75\,mm^2$

Step 7: Permissible Limits for reinforcement.

- $\frac{A_{st,min}}{bd} > \frac{0.85}{f_y} \Rightarrow \frac{A_{st,min}}{400 \times 800} > \frac{0.85}{500}$

$A_{st,min} > 544\ mm^2$

$$\bullet\ A_{st} \leq \text{Minimum} \begin{cases} \text{i) } A_{st,lim} = 0.414\left(\dfrac{f_{ck}}{f_y}\right) \cdot x_{u,lim} \cdot b \\ \quad = 0.414 \times \dfrac{30}{500} \times 0.46 \times 800 \times 400 \\ \quad A_{st,lim} = 3656.45\ mm^2 \\ \text{ii) } 0.04bD = 0.04 \times 400 \times 800 = 14000\ mm^2. \end{cases}$$

$A_{st} \leq 3656.45\ mm^2.$

$$\boxed{\text{Providing } 4\text{-}32\phi + 1\text{-}16\phi}$$

Step 8: Side Face reinforcement: $A_{st} = 0.1\%\ bD$

$A_{st} = 0.001 \times 400 \times 875$

$A_{st} = 350\ mm^2$

Atleast 2-bars on each side face are required to satisfy maximum spacing criteria (300mm)

Providing 2-12 ϕ on each side face.

Step 9: Deflection Check:

$$K_1 = \frac{10}{11.45} \Rightarrow K_1 = 0.87$$

$K_2 \Rightarrow$ Depends on % of Tension reinforcement

$$P_t = \frac{A_{st}}{bD} \times 100$$

$$= \frac{4 \times \frac{\pi}{4} \times 32^2 + 1 \times \frac{\pi}{4} \times 16^2}{400 \times 875} \times 100$$

$P_t = 1.06\%$

$$f_s = 0.58 f_y \frac{A_{st,req}}{A_{st,provided}}$$

$$= 0.58 \times 500 \times \frac{3308.75}{4 \times \frac{\pi}{4} \times 32^2 + 1 \times \frac{\pi}{4} \times 16^2}$$

$f_s = 280.72\ N/mm^2$

$K_2 = 0.85$ .... (from graph 4 of IS456 $P_t = 1.06\%$ & $f_s = 280.72\ N/mm^2$)

$K_3 = 1$ .... (Singly reinforced)

$K_4 = 1$ ..... (Rectangular)

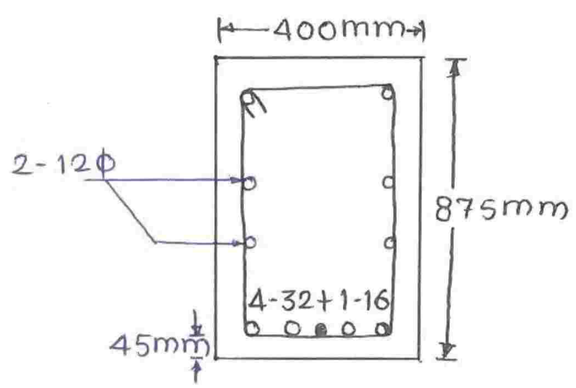
Now,

$$\frac{L_{eff}}{d} < K_1 K_2 K_3 K_4 \text{ (value)}$$

$$\frac{11.45 \times 10^3}{800} < 0.87 \times 0.85 \times 1 \times 1 \times (20)$$

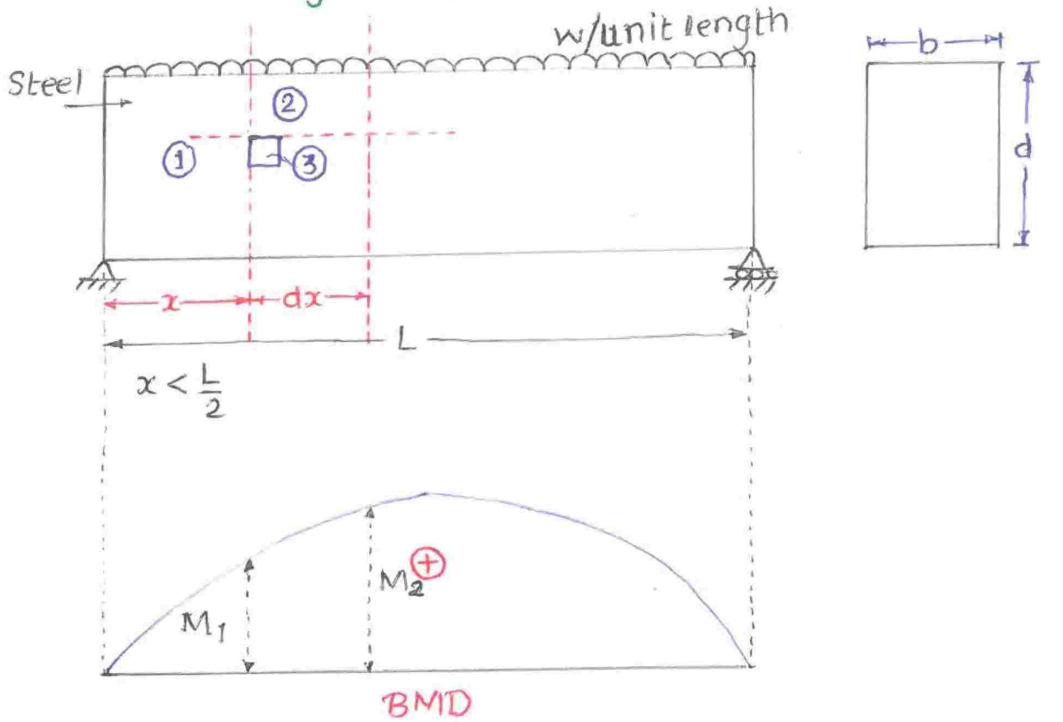
$$14.31 < 14.79 \quad \ldots\ \text{OK.}$$

Detailing:

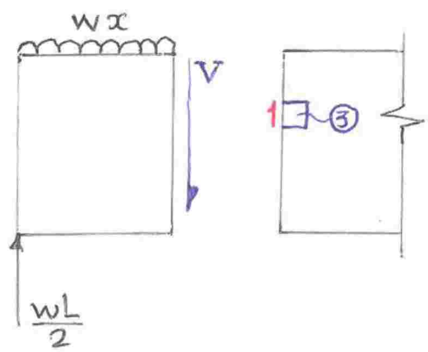


7. Shear

7.1 Shear stress variation over Rectangular Section of Linearly Elastic Homogeneous material :

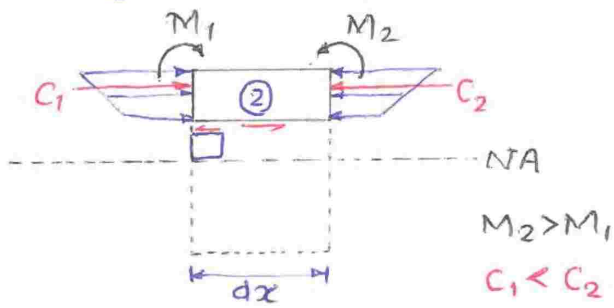


FBD of ①:



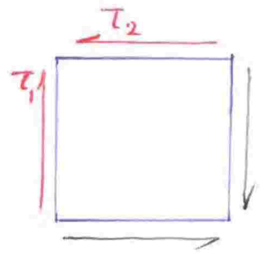
Only vertical equilibrium is represented

FBD of ②



Only horizontal equilibrium is represented.

FBD of ③:



For rotational equilibrium,

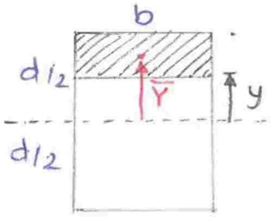
$$\tau_1 = \tau_2$$

We are interested in calculation of τ_1 since, $\tau_1 = \tau_2$ τ_2 is being calculated as follows -

for horizontal equilibrium of ②

$$C_1 - C_2 + \tau_2 (b \cdot dx) = 0$$

$$\tau_2 = \frac{VA\bar{Y}}{Ib}$$



$$\tau_{2y} = \frac{VA\bar{Y}}{Ib}$$

V = SF on section

A = Area of hatched portion

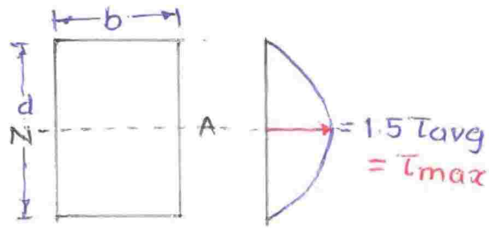
$\bar{Y}$ = distance of C.G. of hatch portion from NA

I = MI of section

b = width of section at level 'y'

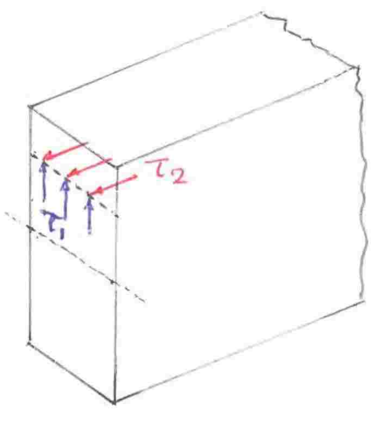
$$\tau_{2y} = \frac{V \cdot \left[b \cdot \left(\frac{d}{2} - y\right)\right] \times \left[\frac{d}{2} - \left(\frac{d/2 - y}{2}\right)\right]}{\frac{bd^3}{12} \times b}$$

$$\tau_{2y} = \frac{6V}{bd^3}\left[\left(\frac{d}{2}\right)^2 - y^2\right]$$

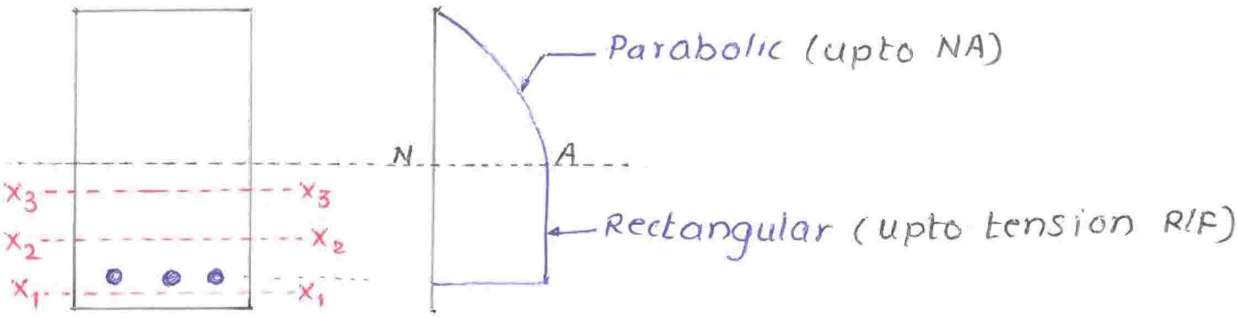


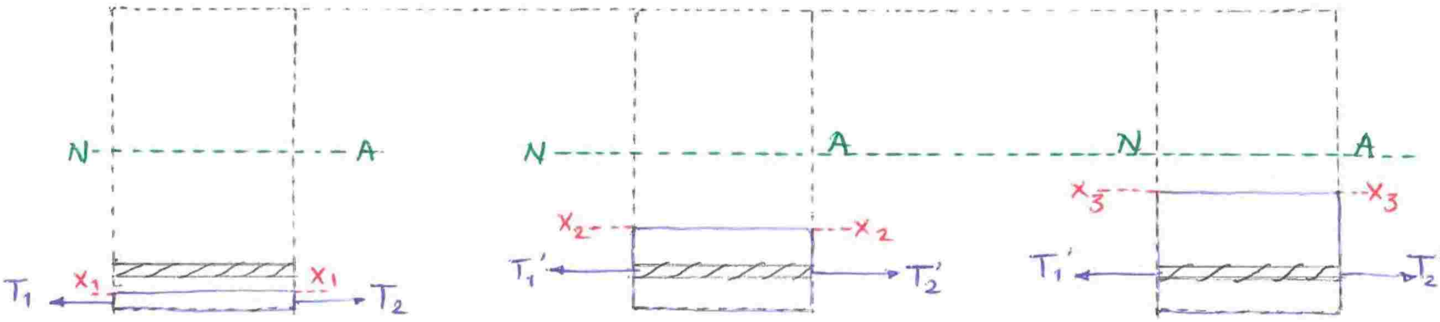
At $y = \pm \frac{d}{2}$, $\tau_{2y} = 0$

at $y = 0$, $\tau_{2y} = \frac{3}{2}\frac{V}{bd} = 1.5\,\tau_{avg}$



7.2 Shear-stress Variation over Rectangular Section of RCC:





Concrete below NA is cracked so, $T_1 = T_2 = 0$. It means shear stress is zero

Concrete below NA is cracked so, entire force will be taken by steel only. It means difference of T_2 & T_1 below (NA) $x_2 - x_2$ and $x_3 - x_3$ remains constant. So shear stress also remains constant.

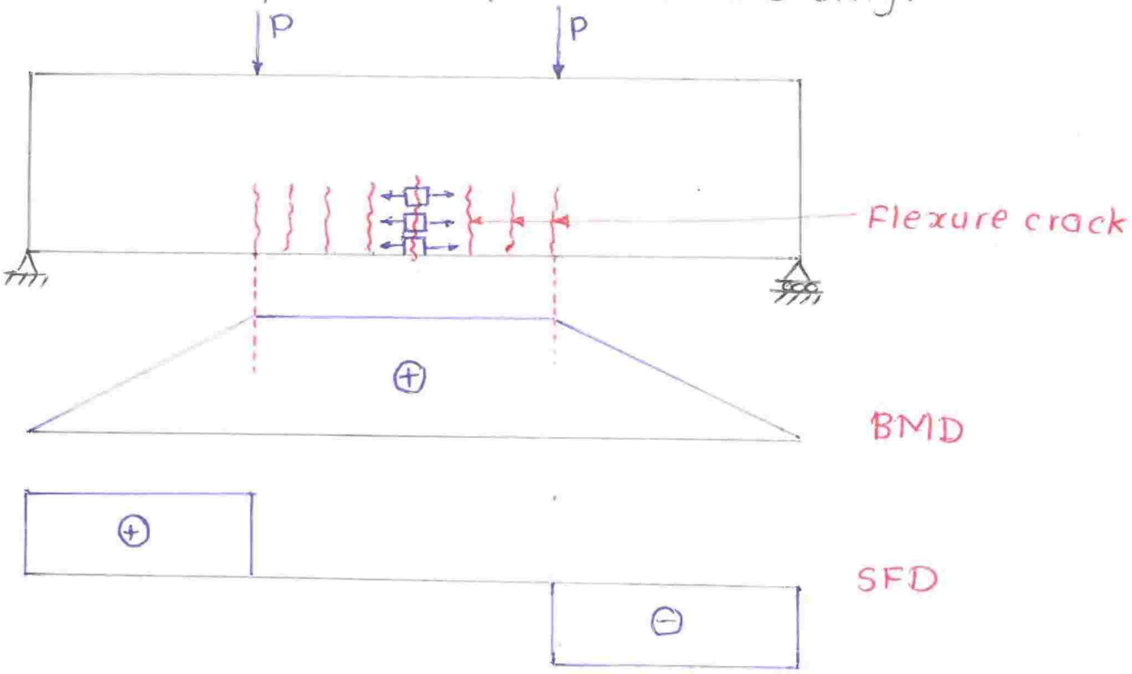
*** Conclusion:**

Shear stress variation over rectangular section of RCC beam subjected to sagging BM is parabolic from top fibre to NA and remains constant upto tension reinforcement.

7.3 Types of Crack :

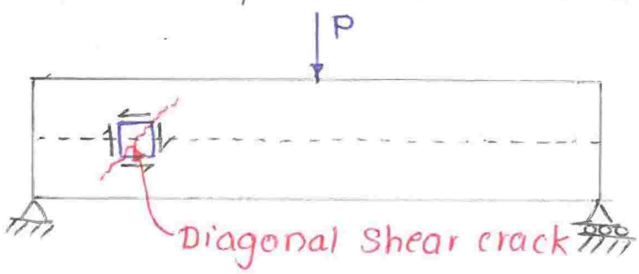
7.3.1 Flexure Crack :

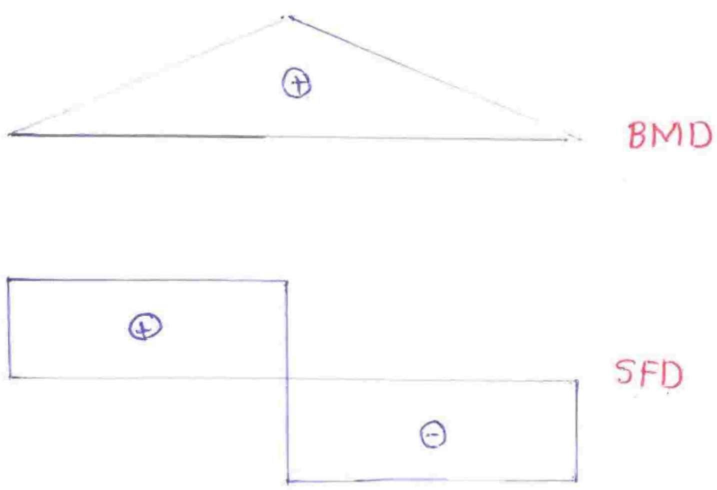
It develops due to flexure stress only.

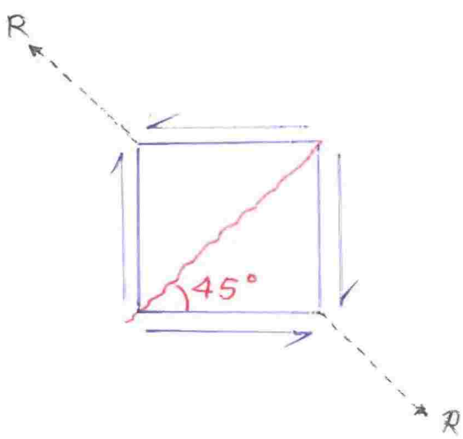


7.3.2 Shear Crack :

It develops due to shear stress only.

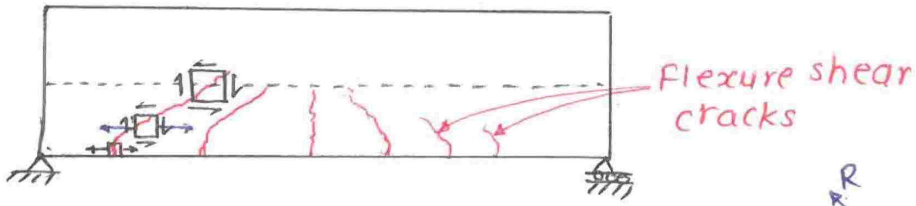


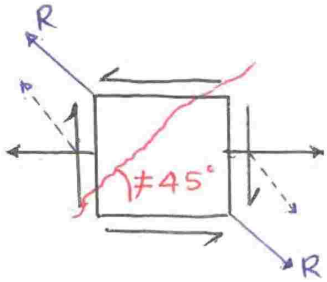


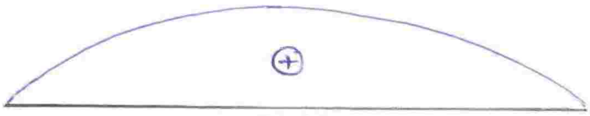


7.3.3 Flexure Shear Crack:

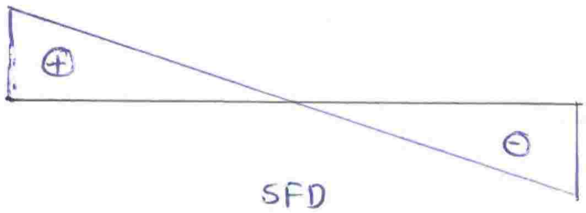
It develops due to combined effect of flexure and shear stress.





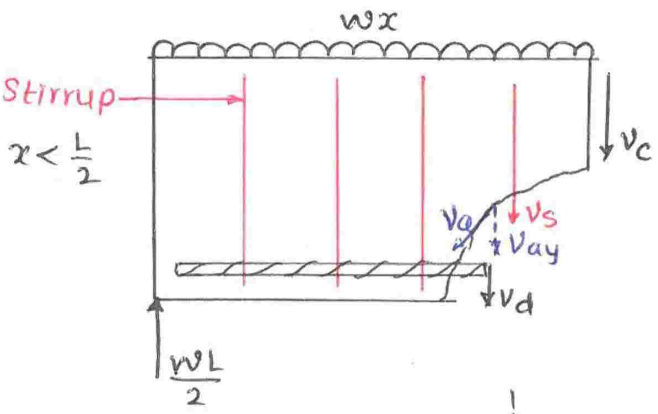


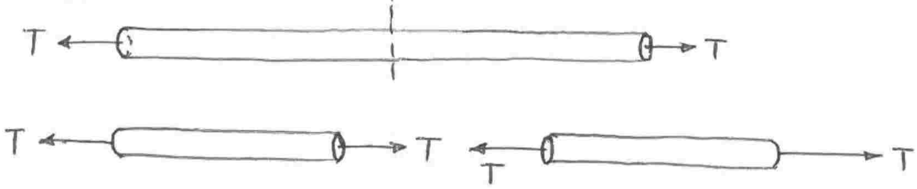
BMD



SFD

7.4 Shear Transfer Mechanism:





All forces are away from section.

Total Shear Resistance of section $= (V_c + V_{ay} + V_d) + V_s$.

V_c = Shear resistance offered by uncracked concrete.

V_{ay} = Vertical component of resistance offered by aggregate interlocking.

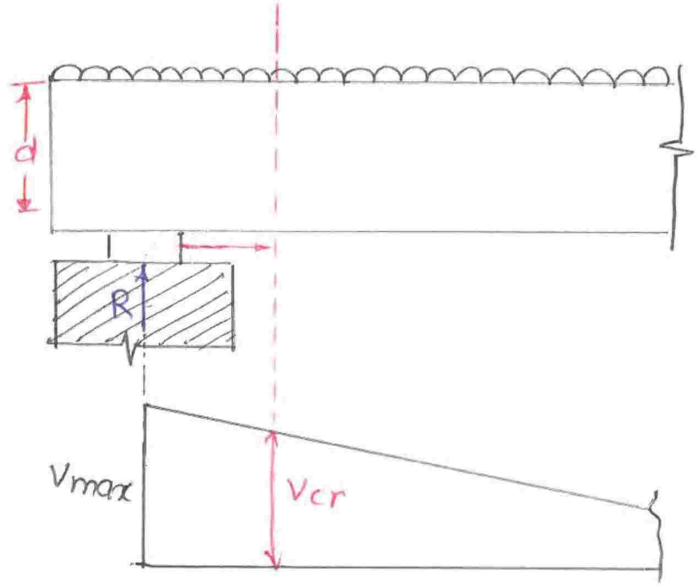
V_d = Shear Resistance offered by longitudinal tension RIF by dowel action

V_s = Shear resistance offered by shear reinforcement.

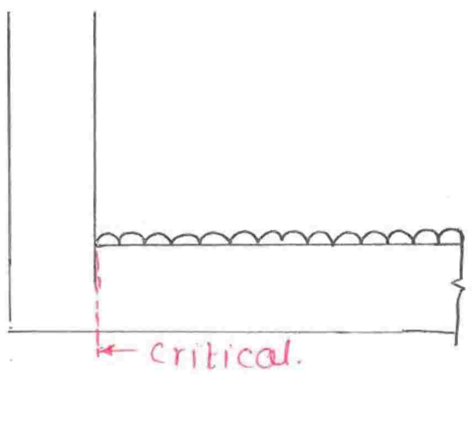
Combined effect of first 3-components $(V_c + V_{ay} + V_d)$ is shear resistance of section without shear reinforcement. Its value is $\tau_c bd$ where, τ_c is design shear strength of concrete and its value is given in Table 19 of IS 456 (Pg 73) corresponding to grade of concrete and % of longitudinal tension reinforcement.

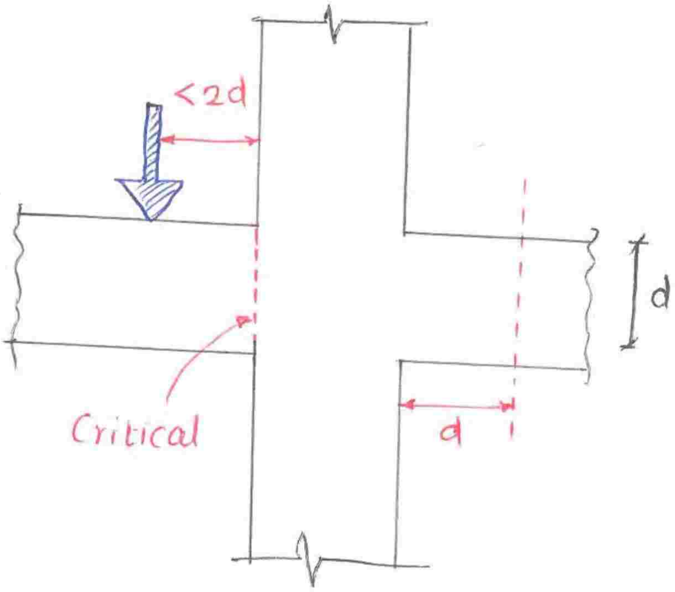
7.5 Critical Section for Shear.

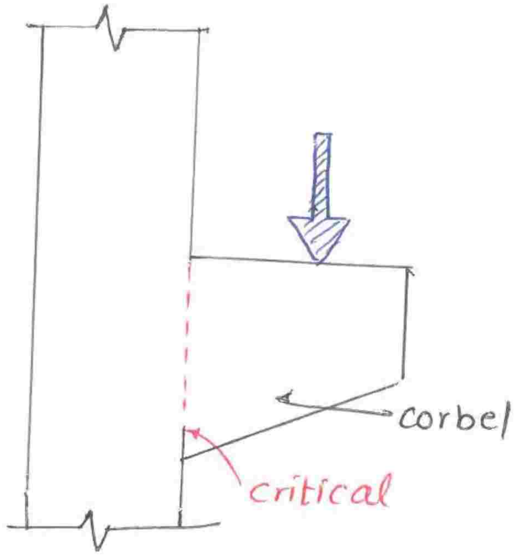
It is that section where chance of failure due to shear is maximum.



If support reaction provides compression to the end of member then critical section for shear is at a distance 'd' from face of of support.







7.6 Nominal Shear Stress:

IS 456 provides uniform stress & Shear Stress distribution instead of as disscussed in 7.2

7.6.1 Beam of Uniform Depth:

$$\text{Nominal Shear Stress} = \tau_v = \frac{V_u}{bd}$$

7.7 Design of section for shear.

Step 1: Calculate design/ factored/ ultimate shear force at critical section.

Step 2: Calculate Nominal shear stress.

$$\tau_v = \frac{V_u \pm \frac{M_u}{d}\tan\beta}{bd} \leq \tau_{c,max}$$

Where $\tau_{c,max}$ is maximum shear st capacity of section with shear reinforcement its value is given in Table 20 of IS 456 (Page 73) corresponding to grade of concrete.

$$\tau_{c,max} \approx 0.63\sqrt{f_{ck}} \ \ N/mm^2.$$

If $\tau_v > \tau_{c,max}$ then section size is increased.

Step 3: Take value of of τ_c from Table 19 of IS 456. corresponding to grade of concrete and % of tension reinforcement.

τ_c is modified as $\delta\tau_c$ for member subjected to axial compression.

$$\delta = \text{Minimum} \begin{cases} \text{i)} \ 1 + \dfrac{3P_u}{f_{ck} A_g} \\ \text{ii)} \ 1.5 \end{cases}$$

τ_c is modified as $k\tau_c$ for slab thickness

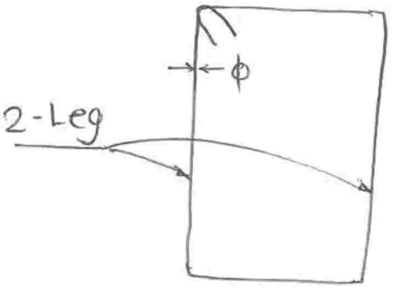
$$k = 1.3 \qquad D \leq 150mm$$

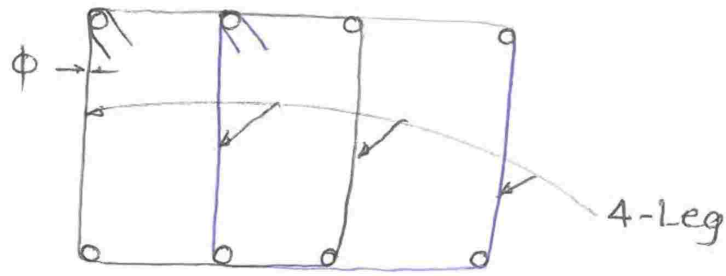
$$= 1.6 - 0.002D \qquad 150 < D < 300$$

$$= 1.0 \qquad 300mm \leq D$$

Step 4: If $\tau_v < \frac{\tau_c}{2}$ then nominal shear stirrup is provided in primary members and no shear stirrup is provided in member of minor importance. (lintel)

$$\boxed{\frac{A_{sv}}{b s_v} \geq \frac{0.4}{0.87 f_y}}$$





$$A_{sv} = 2 \times \left(\frac{\pi}{4} \times \phi^2\right) \qquad A_{sv} = 4 \times \left(\frac{\pi}{4} \times \phi^2\right)$$

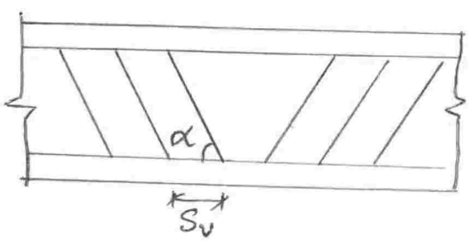
If $\frac{\tau_c}{2} \leq \tau_v < \tau_c$ then nominal shear stirrup is provided in all types of member.

$$\frac{A_{sv}}{b S_v} \geq \frac{0.4}{0.87 f_y}$$

If $\tau_c \leq \tau_v \leq \tau_{c,max}$ then shear reinforcement is designed for shear force $(\tau_v - \tau_c) bd$

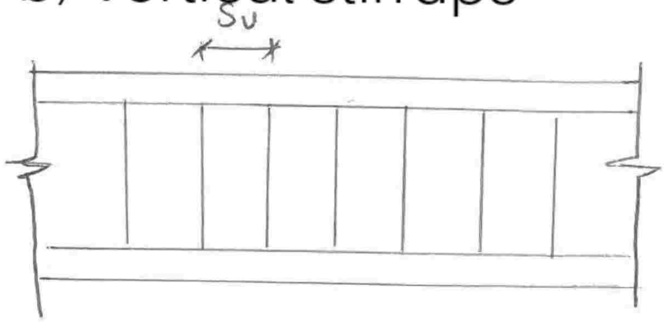
Step 5: Shear reinforcement is provided by following three ways.

1. Inclined Stirrup.



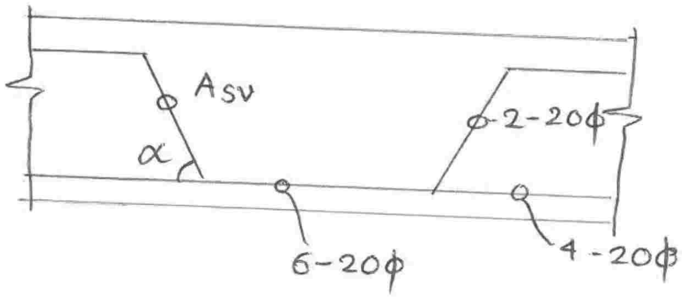
$$V_{us} = \frac{0.87 f_y A_{sv} d}{S_v} (\sin\alpha + \cos\alpha)$$

b) vertical stirrups



$$V_{us} = \frac{0.87 f_y A_{sv} d}{S_v}$$

③ Bent-up Bars :



$$V_{us} = 0.87 f_y \cdot A_{sv} \cdot \sin\alpha$$

*Note:-

- Not more than 50% of shear force taken by shear reinforcement $[(\tau_v - \tau_c)bd]$ is assumed to be taken by bent ~~b~~up bars

  Bent up bars are always provided with shear stirrups so shear stirrups are designed for maximum of following shear forces.

  - $\frac{(\tau_v - \tau_c)bd}{2}$
  - $(\tau_v - \tau_c)bd$ - Capacity of bent-up bars.

	Case I	Case II
V_u	200 kN	200 kN
V_c	30 kN	30 kN
Shear force taken by Shear R/F	200 − 30 = 170 kN	170 kN
Capacity of bent-up bars	250 kN	40 kN
Shear force taken by Stirrup	$\frac{170}{2} = 85$ kN	170 − 40 = 130 kN

- ~~α sh~~ $\boxed{\alpha \not< 45^\circ}$ (preferably $45^\circ - 60^\circ$)

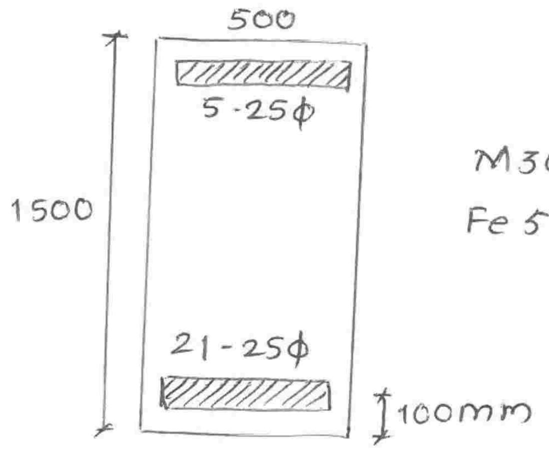
- f_y is not assumed to be more than 415 N/mm^2, irrespective of grade of steel. This limitation is imposed to control crack width.

  - Fe 250 ⟶ 250 N/mm^2
  - Fe 415 ⟶ 415 N/mm^2
  - Fe 500 ⟶ 415 N/mm^2.

Step 6: Maximum permissible spacing of stirrups.

$S_v \leq$ Minimum
- as per step 5
- $\frac{A_{sv}}{b S_v} \geq \frac{0.4}{0.87 f_y}$
- 0.75d and d for vertical & inclined stirrup respectively
- 300 mm

* Ex. Design 4-legged vertical stirrups for section given below which is subjected to ultimate shear force 1000 kN.



M30

Fe 500.

⇒

Step 1: $V_u = 1000\,kN$

Step 2 $\tau_v = \dfrac{V_u}{bd} = \dfrac{1000 \times 10^3}{500 \times 1400}$

$\tau_v = 1.43\ N/mm^2\ (< \tau_{max} = 3.5\ N/mm^2)$

Step 3: $P_t = \dfrac{A_{st}}{bd} \times 100$

$= \dfrac{21 \times \frac{\pi}{4} \times 25^2 \times 100}{500 \times 1400}$

$P_t = 1.47\%$ ⇒ $\tau_c = 0.75\ N/mm^2$ [Table 19 of IS 456 $P_t = 1.47\%$ & M30]

Step 4: Since $\tau_v > \tau_c$ so shear R/F is designed for

$SF = (\tau_v - \tau_c)\, bd$

$= (1.43 - 0.75) \times 500 \times 1400$

$SF = 476\ kN.$

Step 5: $\phi\ (\uparrow) \longrightarrow S_v\ (\uparrow)$

No. of legs $(\uparrow) \longrightarrow S_v\ (\uparrow)$

Assuming 4-legged - 8 φ

$V_{us} = \dfrac{0.87\, f_y\, A_{sv}\, d}{S_v}$

$$476 \times 10^3 = \frac{0.87 \times 415 \times 4 \times \frac{\pi}{4} \times 8^2 \times 1400}{S_v}$$

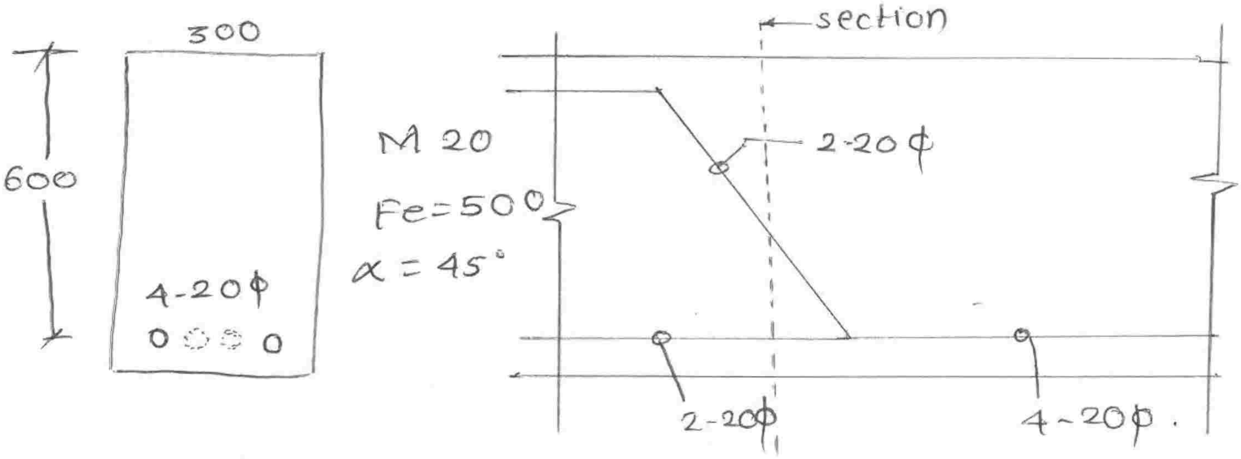
$$S_v = 213.5 \text{ mm}$$

Step 6: Spacing:

$$S_v \leq \text{Minimum} \begin{cases} \bullet\ 213.5 \text{ mm} \\ \bullet\ \frac{A_{sv}}{b s_v} \geqslant \frac{0.4}{0.87 f_y} \Rightarrow S_v \leq 362.96 \text{ mm} \\ \bullet\ 0.75d = 1050 \text{ mm} \\ \bullet\ 300 \text{ mm} \end{cases}$$

Providing 4-legged 8 mm ϕ @ 200 c/c

*Ex. Design shear reinforcement for section given below which is subjected to working load shear force 200 kN. Assume 2-bars used for bent up bars.



$\Rightarrow$

Step 1: $V_u = 1.5 \times 200 = 300 \text{ kN}$

$$\tau_v = \frac{V_u}{bd} = \frac{300 \times 10^3}{300 \times 600}$$

$$\tau_v = 1.67 \text{ N/mm}^2 < \tau_{c,max} \ (2.8 \text{ N/mm}^2)$$

Step3 $P_t = \frac{A_{st}}{bd} \times 100$

$= \frac{2 \times \frac{\pi}{4} \times 20^2}{300 \times 600} \times 100$

$P_t = 0.35\%$

(from Table 19 $P_t = 0.35\%$ & M20)

87 $\tau_c = 0.39 \ N/mm^2$

Step4: Since $\tau_v > \tau$ so section is designed for

$SF = (\tau_v - \tau_c) \cdot bd$

$= (1.67 - 0.39) \times 300 \times 600$

$SF = 230.4 \ kN$

Step5: Capacity of bent-up bars

$= 0.87 f_y \cdot A_{sv} \cdot \sin\alpha$

$= 0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 20^2 \times \sin 45°$

$= 160.41 \ kN$

Bent-up bars are always provided with stirrups. so, stirrups should be designed for maximum of following.

- $\frac{(\tau_v - \tau_c) bd}{2} = \frac{230.4}{2} = 115.2 kN$
- $(\tau_v - \tau_c) bd$ − capacity of bent up bars $= 230.4 - 160.4$ $= 69.99 kN$

Assuming 2-legged 8 ϕ

$V_{us} = \frac{0.87 f_y \cdot A_{sv} \cdot d}{S_v}$

$\Rightarrow S_v = \frac{0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 20^2 \times 600}{115.2 \times 10^3}$

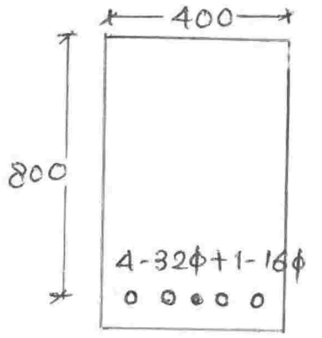
$S_v = 189.04 \ mm$

Step 6 :

$S_v \leq$ Minimum
- 189.04 mm
- $\dfrac{A_{sv}}{bS_v} \geq \dfrac{0.4}{0.87 f_y} \Rightarrow S_v \leq 302.4\,mm$
- $0.75d = 0.75 \times 600 = 450\,mm$
- 300 mm

$\Rightarrow$ Providing 2-legged 8ϕ @ 175 mm c/c.

* Ex. Design shear reinforcement for the beam given in ex. of chapter 6. Assume 2-32ϕ bars are curtailed before critical section for shear.



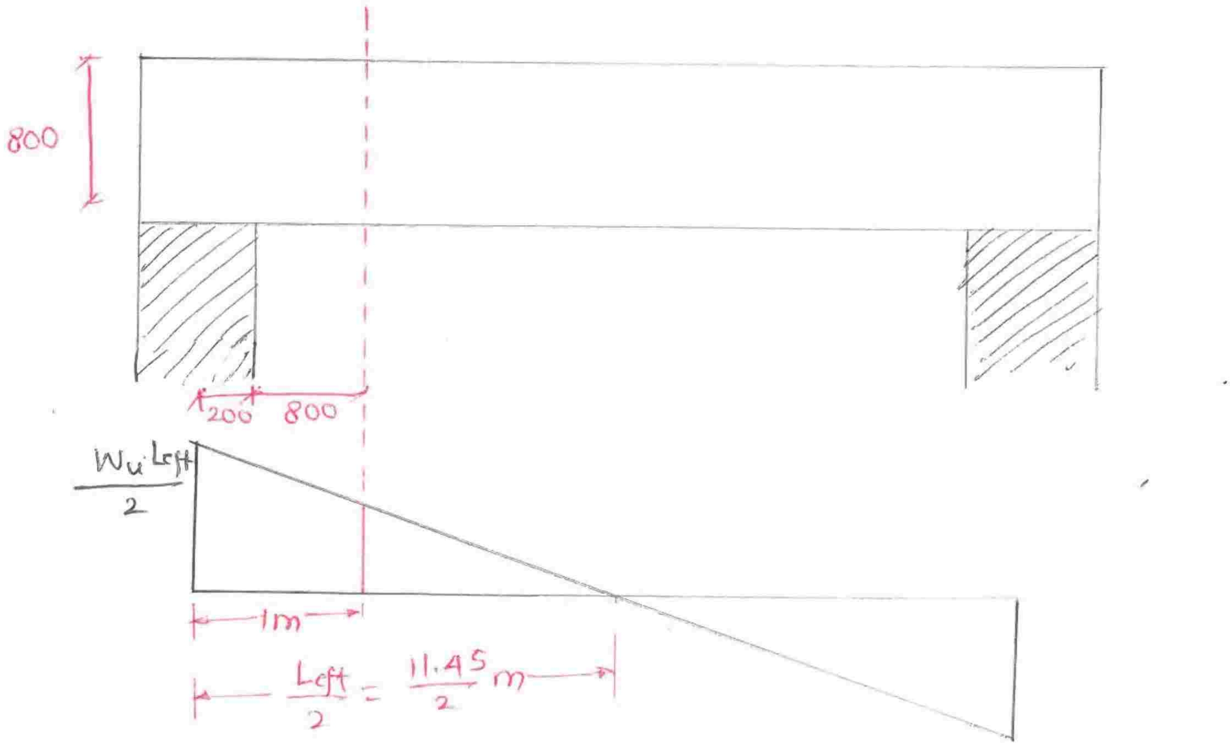
M30

Fe500

$W_u = 58.125\,kN/m$

$L_{eff} = 11.45\,m$

Support width = 400 & 500 mm.



$V_{cr}(x=1m)$ $\quad 2$

$$= \frac{58.125 \times 11.45}{2} - 58.125$$

$$V_{cr} = 274.64\ kN$$

Step 2: $\quad \tau_v = \frac{V_u}{bd} = \frac{274.64 \times 10^3}{400 \times 800} = 0.85\ N/mm^2 < \tau_{c,max}$ $(3.5\ N/mm^2)$

Step 3: $\quad P_t = \frac{A_{st}}{bd} \times 100$

$$= \frac{2 \times \frac{\pi}{4} \times 32^2 + 1 \times \frac{\pi}{4} \times 16^2}{400 \times 800} \times 100$$

$P_t = 0.56\%$ $\quad \left[\text{from Table 19 of IS456} \quad P_t = 0.56\% \ \& \ M30 \right]$

Step 4: Since $\tau_v > \tau_c$ so shear reinforcement is designed for $SF = (\tau_v - \tau_c)bd$

$$= (0.85 - 0.52) \times 400 \times 800$$

$$SF = 105.6\ kN$$

Step 5: Assuming 2-legged 10ϕ

$$V_{us} = \frac{0.87 f_y A_{sv} \cdot d}{S_v}$$

$$105.6 \times 10^3 = \frac{0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 10^2 \times 800}{S_v}$$

$$\Rightarrow S_v = 429.6\ mm$$

* Note:

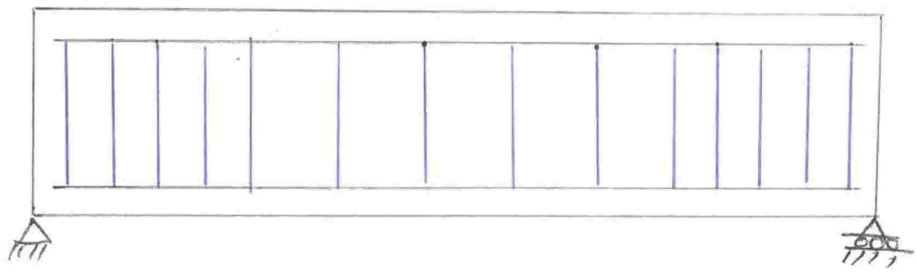
8 mm dia should have taken for more appropriate result.

Step-5: Spacing:

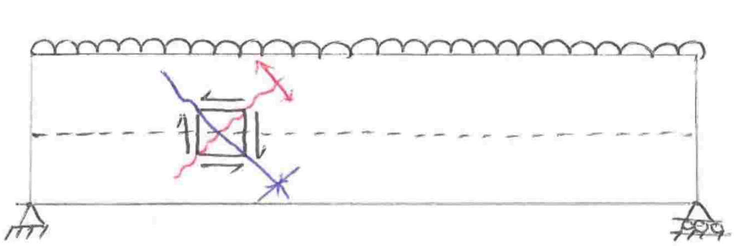
$$S_v = \text{Minimum} \begin{cases} \bullet\ 429.6\,mm \\ \bullet\ \dfrac{A_{sv}}{bS_v} \geq \dfrac{0.4}{0.87f_y} \Rightarrow S_v \leq 354.46\,mm \\ \bullet\ 0.75d = 0.75 \times 800 = 600\,mm \\ \bullet\ 300\,mm \end{cases}$$

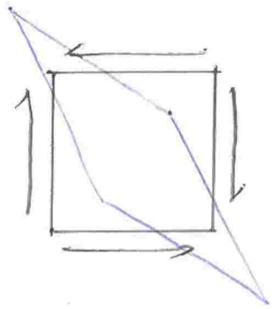
Providing 2-legged 10ϕ @ 300 mm c/c

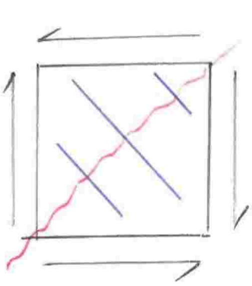
* Note:
Spacing of stirrups is increased in middle portion because shear force is less as compared to supports.



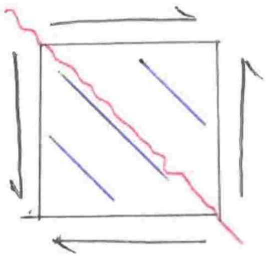
7.8 Reason of $\tau_{c,max}$:



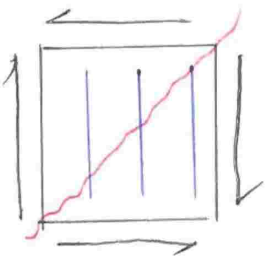




(a) Inclined Stirrups



(b) Reversal of stress



c) Vertical stirrups

8. Bond & Detailing

8.1 Introduction:

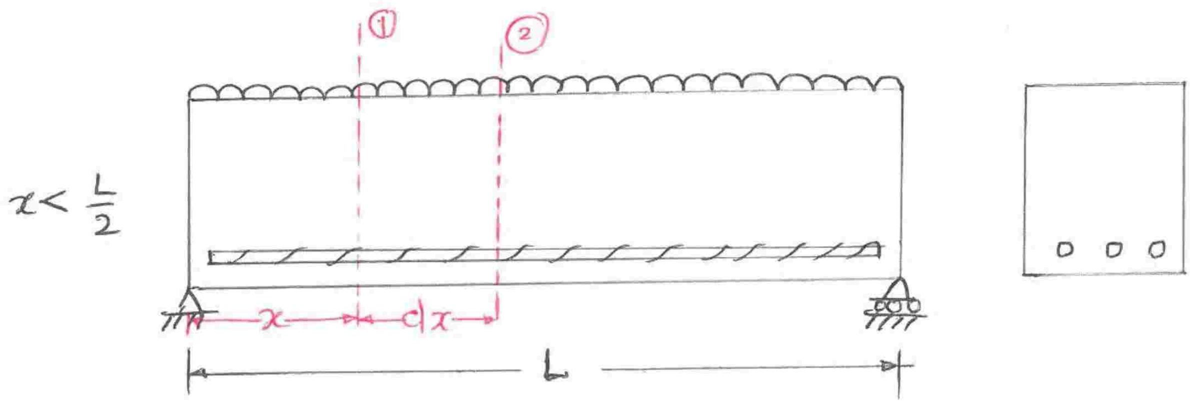
Sufficient bond is required between reinforcement and concrete to prevent relative movement and proper detailing of reinforcement is required for full utilisation of member strength without any premature failure.

8.2 Types of Bond:

1. Flexure Bond
2. Anchorage Bond.

8.2.1 Flexure Bond:

It develops due to variation of BM along length of reinforcement.



At section 1-1

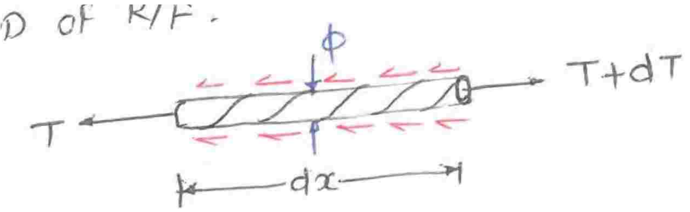
$$M = T \times LA. \quad \cdots\cdots\cdots (i)$$

At section 2-2

$$M + dM = (T + dT) \times LA \quad \cdots\cdots\cdots (ii)$$

From (ii) - (i)

FBD of R/F.



$\Sigma F_x = 0$

$\Rightarrow \cancel{T} - (\cancel{T} + dT) - \tau(n\pi\phi)\cdot dx = 0$

$dT = \tau(n\pi\phi)\,dx \Rightarrow \frac{dM}{LA} = \tau(n\pi\phi)dx$

$\frac{dM}{dx} = \tau(n\pi\phi) \times LA$

$\tau = \frac{dM}{dx} \times \frac{1}{(n\pi\phi)\,LA}$

$\Rightarrow \boxed{\tau = \frac{V}{(n\pi\phi)\,LA}}$

Value of τ calculated above should be less than permissible bond strength (design bond stress) between reinforcement and concrete.

Design bond stress (τ_{bd})

Grade of concrete	M20	M25	M30	M35	M40 & above
τ_{bd} (N/mm²)	1.2	1.4	1.5	1.7	1.9

- Above values are corresponding to plain bar under tension
- Above values are enhanced by 60% for deformed bars.

Plain bar

Deformed bar (HYSD)

- Above values are increased by ~~20%~~ 25% for bar under compression.

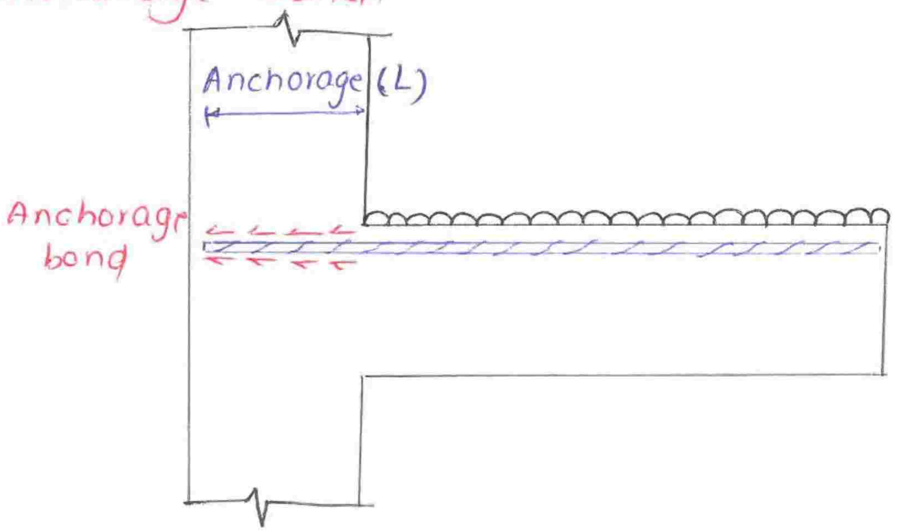
Ex. Calculate design bond stress for bar of Fe415 under compression in M30 concrete.

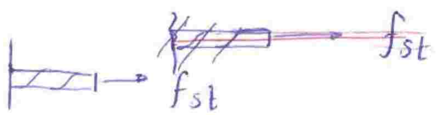
$\Rightarrow$ $\tau_{bd} = 1.5 \times \underset{\text{comp.}}{1.25} \times \underset{\text{HYSD}}{1.6} = 3\ N/mm^2$

*Note:

In case of bond failure, the most economical way to make it safe is by providing more number of bars of smaller dia. instead of less number of bars of larger dia. for same amount of steel.

8.2.2 Anchorage Bond:



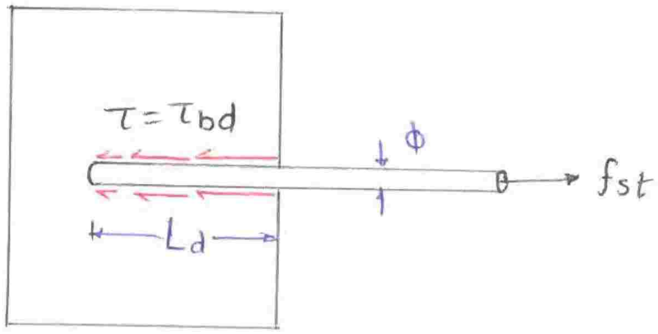


This bond stress develops around bar to provide proper anchorage so that reinforcement attains its desired stress.

In above fig., L is provided for anchorage of reinforcement only so bond stress around the R/F in this length is anchorage bond (no BM variation in this portion)

8.3 Development Length:

Minimum length of reinforcement required to be embeded in concrete for development of permissible stress of reinforcement ($0.87 f_y$)



$$\frac{\pi}{4} \times \phi^2 \times f_{st} = \tau_{bd} (\pi \phi) \cdot L_d$$

$$L_d = \frac{f_{st} \cdot \phi}{4 \tau_{bd}}$$

$$L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}}$$

Ex. Compare L_d of bar of Fe 250 and Fe 415 of same dia and embeded in same concrete.

⇒

- Fe 250

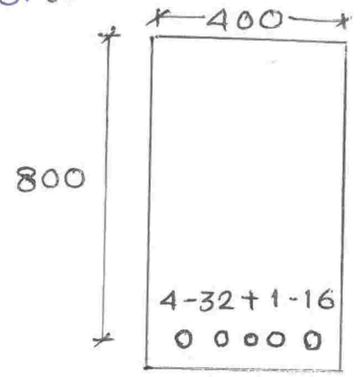
$$L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}} = \frac{0.87 \times 250 \times \phi}{4 \times \tau_{bd}} = \frac{54.37 \phi}{\tau_{bd}}$$

- Fe 415

$$L_d = \frac{0.87 f_y \phi}{4 \times \underset{\uparrow \text{ HYSD}}{1.6} \tau_{bd}} = \frac{0.87 \times 415 \times \phi}{4 \times 1.6 \times \tau_{bd}} = \frac{56.41 \phi}{\tau_{bd}}$$

So development length of HYSD bar is invariably higher than as of mild steel, irrespective of dia of bar and grade of concrete.

Ex. Apply check for bond at simple support in beam of example of chapter 6. Assume 2-32ϕ bars are curtailed before support.



M30, Fe 500

$W_u = 58.125$ kN/m

$L_{eff} = 11.45$ m

Support width = 400mm & 500mm

Severe exposure.

$\Rightarrow$

$$L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}} = \frac{0.87 \times 500 \times 32}{4 \times \underset{\substack{\uparrow \\ \text{HYSD}}}{1.6} \times 1.5}$$

$$L_d = 1450 \text{ mm}$$

M_1 = MR of section at mid of simple support with $2\text{-}32\phi + 1\text{-}16\phi$

For position of N.A

$$C = T$$

$$0.36 f_{ck} x_u b = 0.87 f_y A_{st}$$

$$0.36 \times 30 \times x_u \times 400 = 0.87 \times 500 \times \left(2 \times \frac{\pi}{4} \times 32^2 + 1 \times \frac{\pi}{4} \times 16^2\right)$$

$$x_u = 182.2 \text{mm}$$

$M_1 = C \times LA = 0.36$

V= SF at mid of simple support

$$= \frac{W_u L_{eff}}{2}$$

$$= \frac{58.125 \times 11.45}{2}$$

$V = 332.76$ kN

Now

$$L_d \leq \frac{1.3 M_1}{V} + L_o$$

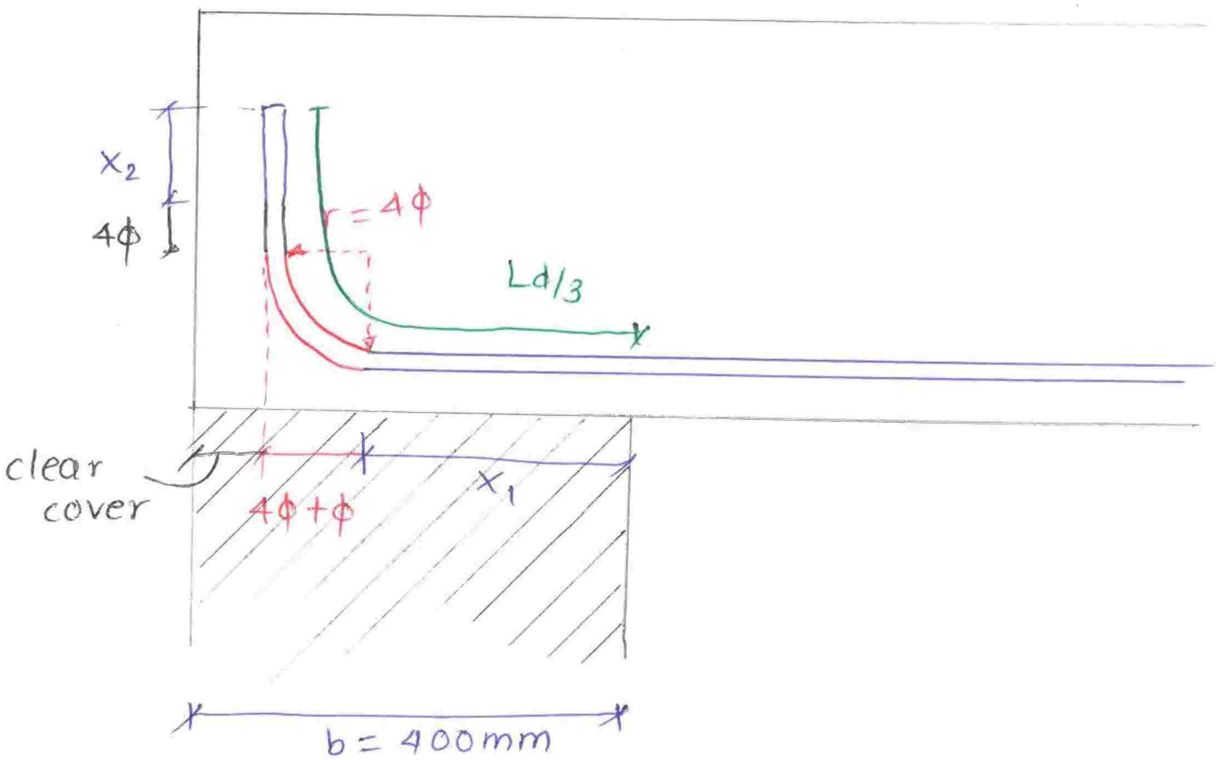
$$1450 \leq \frac{1.3 \times 569.45 \times 10^6}{332.76 \times 10^3} + L_o$$

$$L_o \geq -774.68 \text{ mm}$$

* Note:
Negative value of L_o represents no requirement of extension of reinforcement beyond centre of support.

In addition to above criteria, reinforcement must be extended into the support for a distance not less than

$\frac{L_d}{3} \Rightarrow \quad \frac{L_d}{3} = \frac{1450}{3} = 483.33$ mm



$$\frac{L_d}{3} = (b - \text{clear cover} - 5\phi) + 8\phi + X_2$$

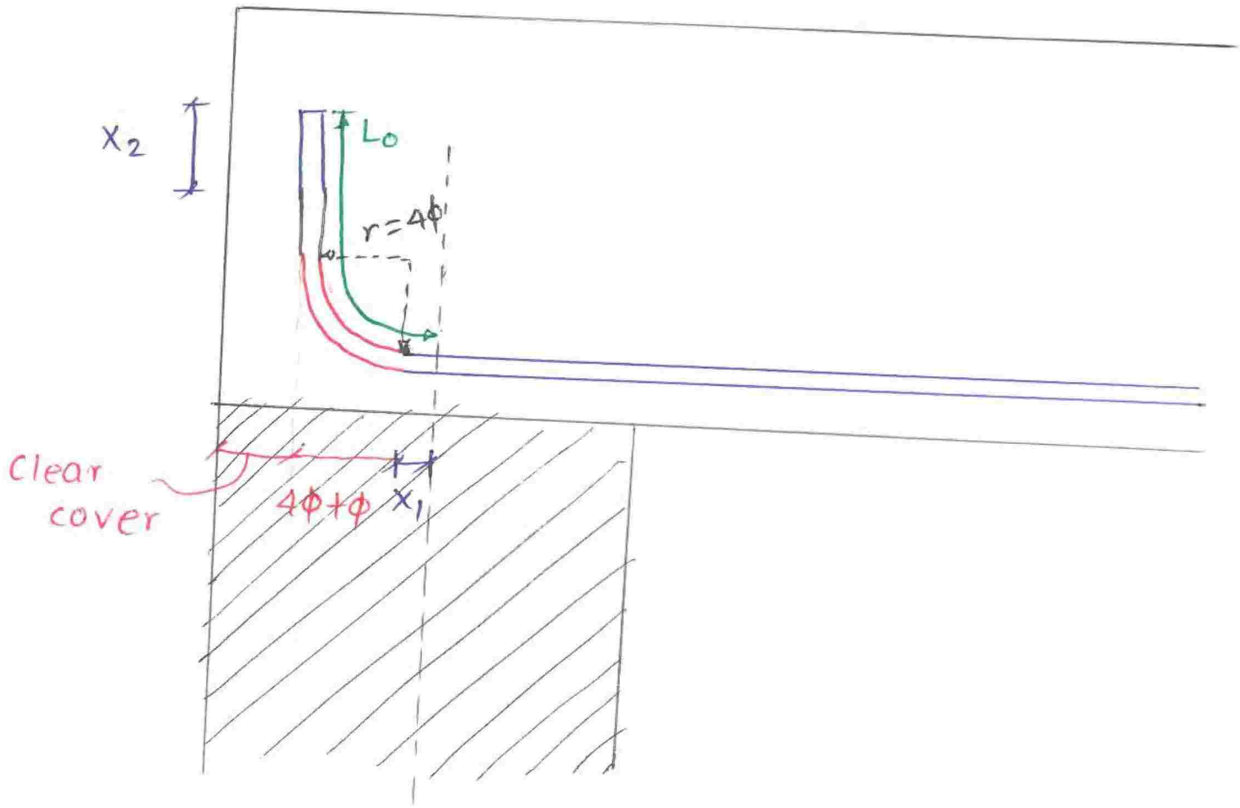
$$483.33 = (400 - \underset{\uparrow\ (\text{Severe})}{45} - 5\times 32) + 8\times 32 + X_2$$

$$X_2 = 32.33\ mm$$

Providing $X_2 = 40mm$

*Note

- For $L_o > 0$, $\frac{L_d}{3}$ and 'b' of above expression are replaced by L_o and $\frac{b}{2}$ respectively.



$$L_o = \left(\frac{b}{2} - \text{clear cover} - 5\phi\right) + 8\phi + X_2$$

10. Compression Member

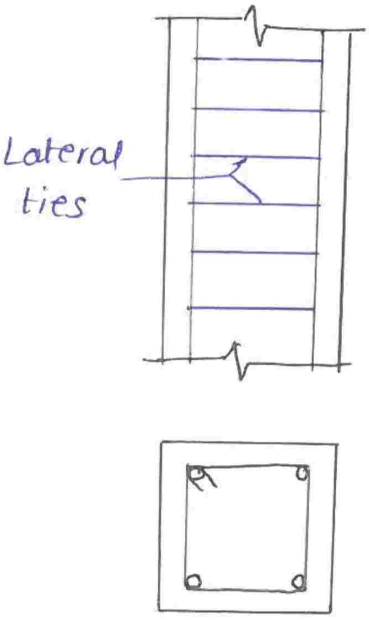
10.1 Introduction:

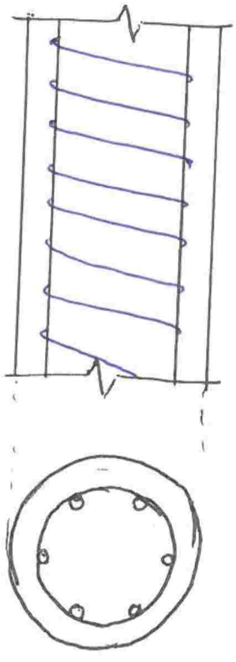
Compression member is a structural member which is primarily subjected to axial compression. If orientation is vertical then termed as column otherwise strut.

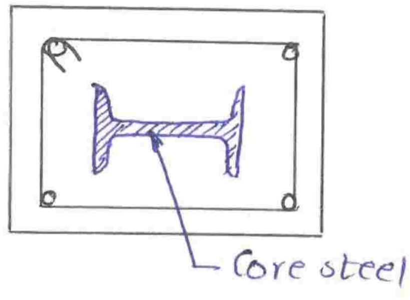
10.2 Classification of Column:

10.2.1 Based on Type of Steel:

1) Tied Column.
2) Spirally / Helically reinforced column.
3) Composite column.







$A_{core} \ngtr 20\% \, A_{gross}$

1) Tied Column

2) Spirally/Helically R/F column

3) Composite Column

10.2.2 Based on Type of Loading.

1) Concentrically Loaded:

Load is placed at C.G. of section.

2) Axially Loaded:

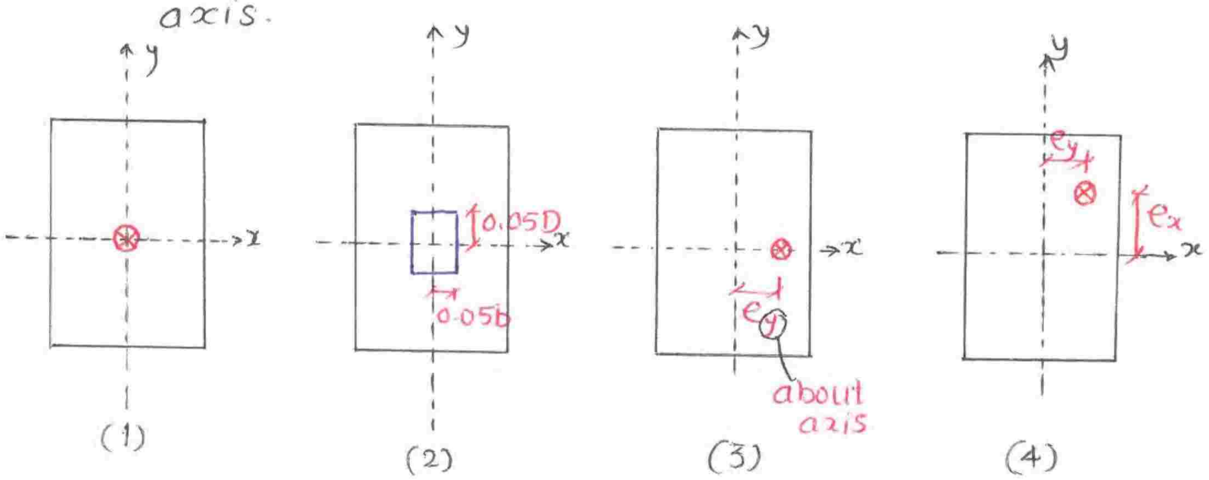
Load is placed at an eccentricity within 5% of lateral dimension.

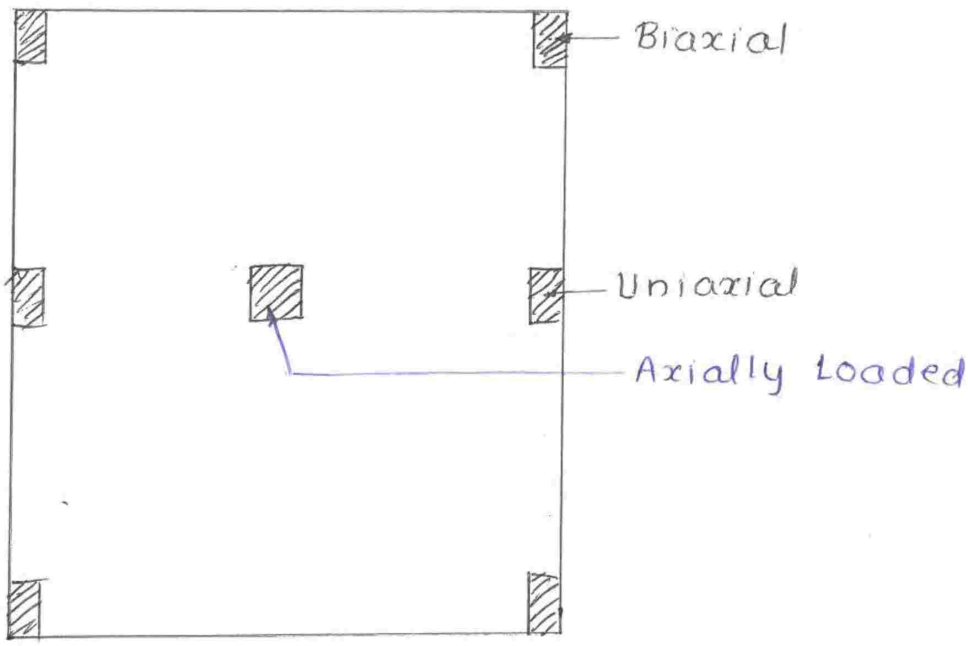
3) Axial Load with uniaxial moment:

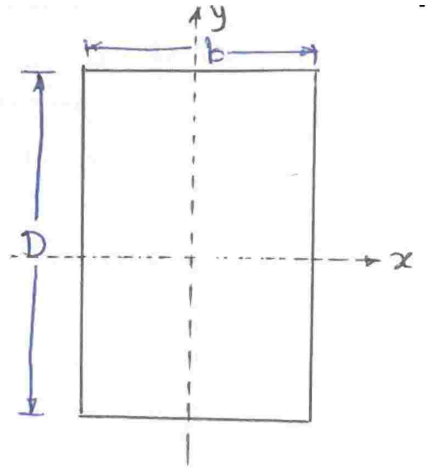
Load is placed at an eccentricity about 1 axis only.

4) Axial load with Biaxial Moment:

Load is placed at an eccentricity about both axis.







$D > b$

$I_{xx} > I_{yy}$

X- axis is considered as Major axis because I_{xx} is greater

Y- axis $\longrightarrow$ Minor axis.

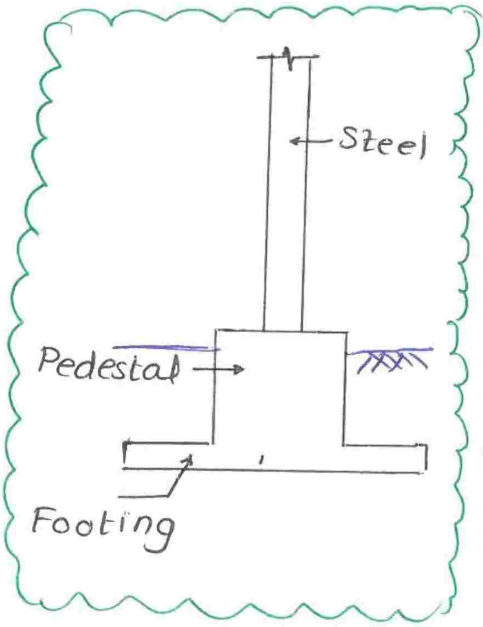
$$\text{Slenderness Ratio} = \frac{L_{eff}}{\text{Lateral Dimension}}$$

$$\lambda_x = \frac{L_{eff,x}}{D}$$

$$\lambda_y = \frac{L_{eff,y}}{b}$$

For $L_{eff,x} = L_{eff,y} = L_{eff}$.

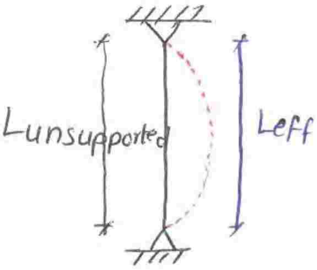
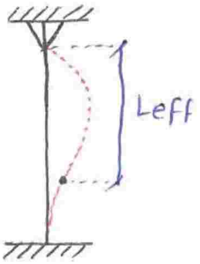
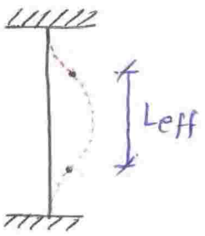
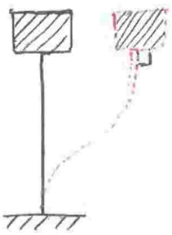
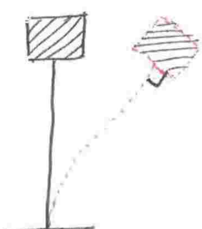
$$\lambda_{max} = \frac{L_{eff}}{\text{Least Lateral Dimension}}$$

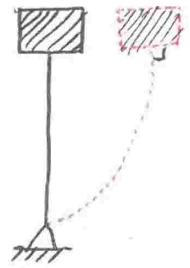


$\lambda_{max} \leq 3$	Pedestal
$3 < \lambda_{max} < 12$	Short Column
$12 \leq \lambda_{max}$	Long Column

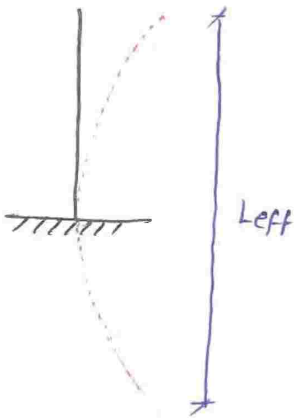
10.5 Effective Length of Column.

It is the distance between point of contraflexure or point of zero moment. In other words, length of column that effectively participates in buckling is called effective length of column.

Support Condition	Theoretical	Reccommended.
	L_{eff} = 1.0 $L_{unsupported}$	L_{eff} = 1.0 $L_{unsupported}$
	L_{eff} = 0.7 $L_{unsupported}$	L_{eff} = 0.8 $L_{unsupported}$
	L_{eff} = 0.5 $L_{unsupported}$	L_{eff} = 0.65 $L_{unsupported}$
	L_{eff} = 1.0 $L_{unsupported}$	L_{eff} = 1.2 $L_{unsupported}$
	–	L_{eff} = 1.5 $L_{unsupported}$

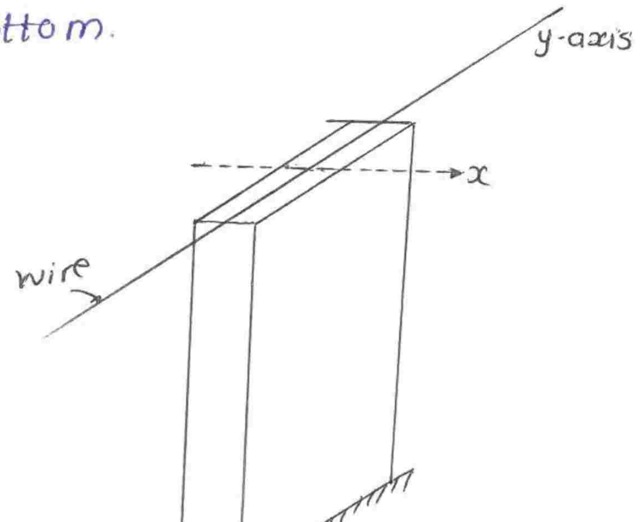


$L_{eff} = 2.0\, L_{unsupported}$ $L_{eff} = 2.0\, L_{unsupported}$.

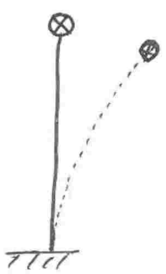


$L_{eff} = 2.0\, L_{unsupported}$ $L_{eff} = 2.0\, L_{unsupported}$.

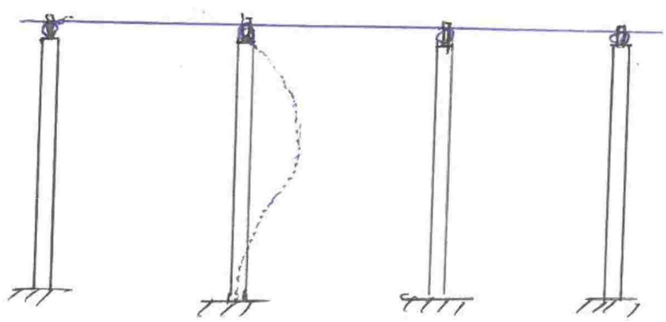
Ex. Calculate effective length of an electric pole carrying wire in one direction at top and embeded in large size foundation at bottom.



About y-axis: About x-axis



$L_{eff} = 2.0L$



$L_{eff} = 0.8L$

$L_{unsupported}$ may also be different about different axes
For ex. a column supported by walls from two opposite sides has different $L_{unsupported}$ about different axes.

10.4 Codal Provisions:

10.4.1 Maximum Permissible Length:

- Laterally restrained at ends

$$L_{unsupported} \not> 60b$$

- Laterally not restrained at ends

$$L_{unsupported} \not> \frac{100b^2}{D}$$

10.4.2 Minimum Eccentricity:

Every column must be designed for minimum eccentricity to account for constructional defects and material imperfection.

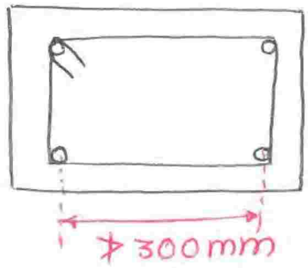
$$e_{min} = \text{Maximum} \begin{cases} \bullet \; \dfrac{L_{unsupported}}{500} + \dfrac{b \text{ or } D}{30} \\ \bullet \; 20\,mm \end{cases}$$

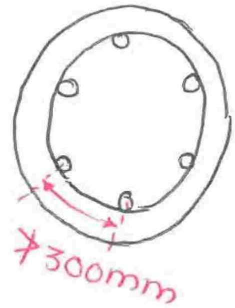
$\perp^{re}$ to axis

10.4.3 Longitudinal Reinforcement:

- Minimum 0.8% of gross area.
- Maximum 6% of gross area (Practically 4% due to lapping)
- Minimum dia. 12mm.
- Minimum 4 bars for rectangular column and 6 for circular column.
- Minimum nominal cover 40mm or dia. of bar whichever is greater. This can be reduced to 25mm for bardia. 12mm and section dimension 200mm or less.

- Minimum 0.15% of gross area for pedestal
- Maximum spacing along peripheri should not exceed 300mm.





10.4.4 Transverse Reinforcement:

- To prevent buckling of longitudinal reinforcement.
- For confinement of concrete.
- To hold longitudinal reinforcement at position.
- To enhance resistance against shear and torsion.

10.4.4.1 Lateral Ties:

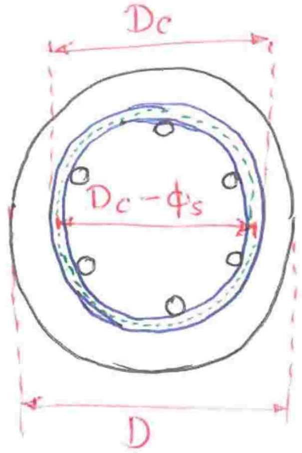
$$\phi \geq \text{Maximum} \begin{cases} \bullet\ \phi_{long,max}/4 \\ \bullet\ 6\,mm \end{cases}$$

$$S \leq \text{Minimum} \begin{cases} \bullet\ \text{Least Lateral dimension} \\ \bullet\ 16\,\phi_{long,min} \\ \bullet\ 300\,mm \end{cases}$$

10.4.4.2 Spiral / Helical Reinforcement:

- Spirally reinforced columns are more ductile and its load carrying capacity is 5% higher than column with lateral ties.
- Concrete of spirally reinforced column is subjected to triaxial compression.
- A column is considered as spirally reinforced if following condition is satisfied.

$$\frac{\text{volume of spiral R/F}}{\text{Volume of core}} \geq \frac{0.36 f_{ck}}{f_y}\left(\frac{A_g}{A_c} - 1\right)$$



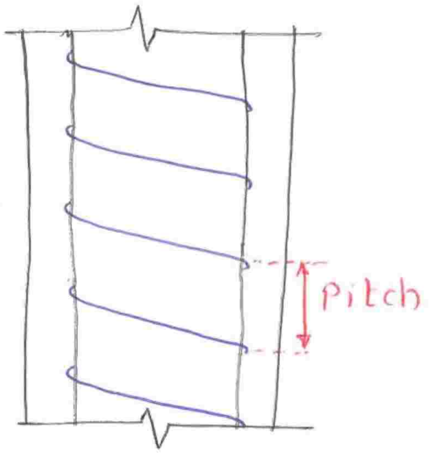
$D_c = D - 2 \times \text{clearcover}$

$A_g = \frac{\pi}{4} D^2$

$A_c = \frac{\pi}{4} D_c^2$

Volume of spiral per pitch

$= \frac{\pi}{4} \phi_s^2 \times \pi (D - \phi_s)$

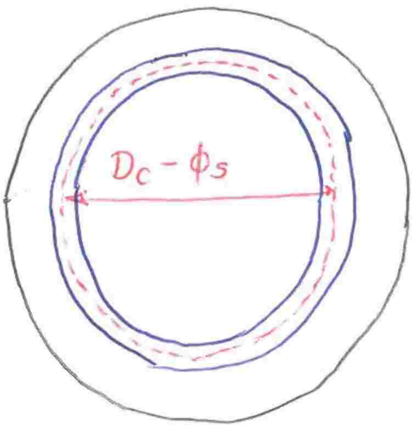


Volume of core per pitch

$= \frac{\pi}{4} D_c^2 \times \text{pitch}$

$\phi_s \geq$ maximum $\begin{cases} \cdot\ \phi_{long,max}/4 \\ \cdot\ 6\text{mm} \end{cases}$

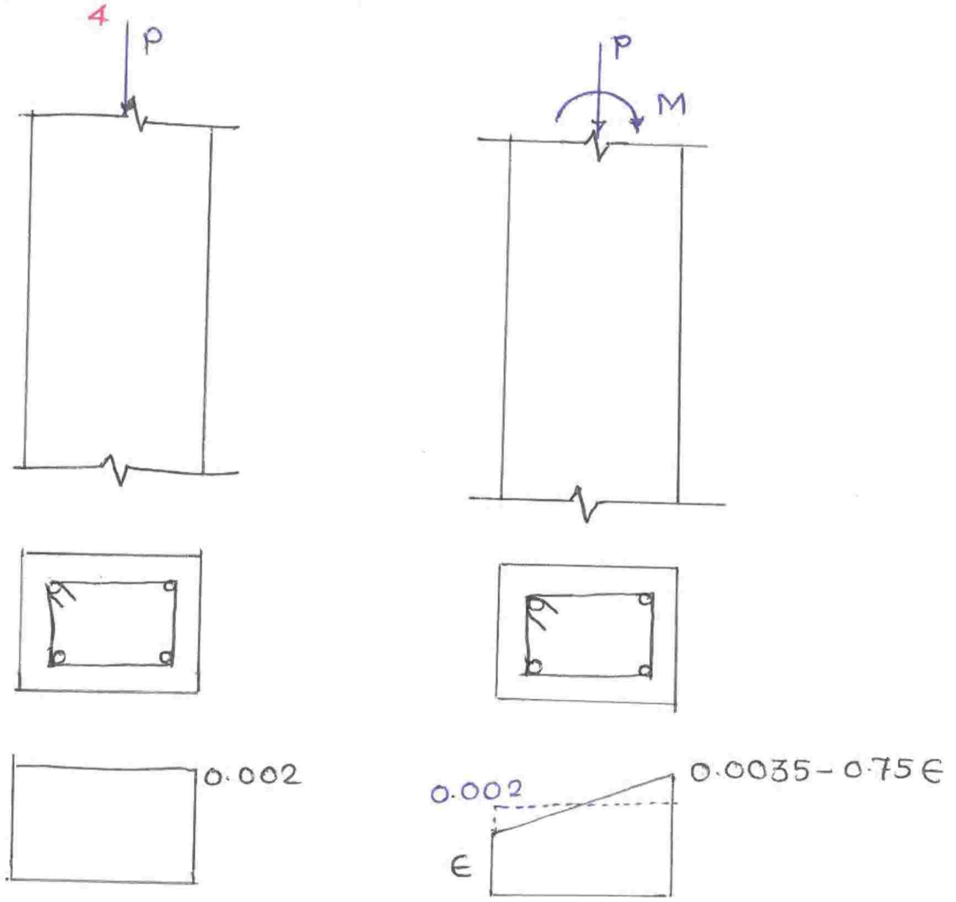
S/pitch $\leq$ Minimum $\begin{cases} \cdot\ 75\text{mm} \\ \cdot\ D_c/6 \end{cases}$



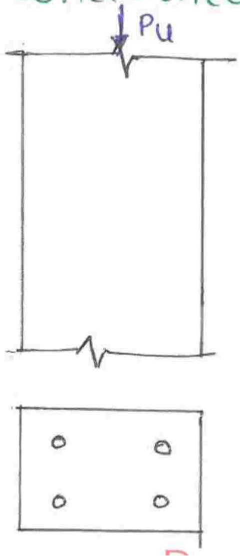
S/pitch $\geq$ Maximum $\begin{cases} \cdot\ 25\text{mm} \\ \cdot\ 3\phi_s \end{cases}$

10.5 Assumptions:

1. Assumption 1 to 5 of limit state of collapse of flexure are applicable for compression member also.
2. For axially loaded column, maximum compressive strain in all fibres is limited to 0.002.
3. For column subjected to axial load with bending and entire section is under compression, maximum strain in concrete is limited to $0.0035 - \frac{3}{4}^{th}$ of compressive strain of least comp. fibre.



10.6 Concentrically loaded Short Column.



$$P_u = P_{uR}$$
$$= P_c + P_s$$
$$= f_c \cdot A_c + f_{sc} \cdot A_{sc}$$
$$= f_c\,(A_g - A_{sc}) + f_{sc} \cdot A_{sc}$$
$$P_u = f_c A_g + (f_{sc} - f_c) \cdot A_{sc}$$

At strain 0.002:-

$f_c = 0.45 f_{ck}$

$f_{sc} = 0.87 f_y$ (Fe 250)

$0.79 f_y$ (Fe415)

$0.745 f_y$ (Fe 500)

Now,

$P_u = 0.45 f_{ck} \cdot A_g + (0.75 f_y - 0.45 f_{ck}) A_{sc}$

For spiral R/F

$P_u = 1.05 [0.45 f_{ck} \cdot A_g + (0.75 f_y - 0.45 f_{ck}) A_{sc}]$

Ex. A RCC short column of size 450 x450mm is reinforced with 4-20φ of Fe 415. calculate concentric working load carrying capacity of column by ignoring reduction of concrete area due to presence of steel. M20 concrete.

⇒ ~~Alex~~

$P_u = 0.45 f_{ck} \cdot A_g + 0.75 f_y \cdot A_{sc}$

$= 0.45 \times 20 \times 450 \times 450 + 0.75 \times 415 \times 4 \times \frac{\pi}{4} \times 20^2$

$P_u = 2213.63 kN$

Working Load $= \frac{P_u}{1.5} = \frac{2213.63}{1.5} = 1475.75 kN.$

10.7 Axially Loaded Short Column:

It is obtained by reducing load carrying capacity of concentrically loaded column by approximately 10% to account for eccentricity of load within 5% of lateral dimension.

$P_u = 0.4 f_{ck} A_g + (0.67 f_y - 0.4 f_{ck}) \cdot A_{sc}$

For spiral R/F

$P_u = 1.05 \times [0.4 f_{ck} A_g + (0.67 f_y - 0.4 f_{ck}) \cdot A_{sc}]$

Ex. Calculate axial load carrying capacity of section 500 x 600mm reinforced with 8-20ϕ of Fe 415. M20 concrete.

⇒

$$P_u = 0.4\, f_{ck}\, A_g + (0.67 f_y - 0.4 f_{ck}) \cdot A_{sc}$$

$$= 0.4 \times 20 \times 500 \times 600$$

$$P_u = 3078.78\ kN$$

10.8 Design of Axially Loaded Short Column:

1. Section size is given and steel is to be designed.
2. Section size and steel both are to be designed.

Ex. Design the reinforcement of a spirally reinforced column of dia. 450mm. It is subjected to factored axial load 3000kN $L_{unsupported}$ 3.4m, pinned supported at both ends M25, Fe 415, Mild exposure.

⇒

Slenderness Ratio, $\lambda = \dfrac{L_{eff}}{D} = \dfrac{3.4 \times 10^3}{450}$

$$\lambda = 7.55 < 12$$

so, column is short

$$e_{min} = \text{Maximum} \begin{cases} \cdot\ \dfrac{L_{unsupported}}{500} + \dfrac{D}{30} = \dfrac{3.4 \times 10^3}{500} + \dfrac{450}{30} = 21.8\,mm \\ \cdot\ 20 \end{cases}$$

$$e_{min} = 21.8\,mm < 0.05D\ (22.5mm)$$

So, column is axially loaded.

Now,

$$P_u = 1.05\,[0.4 f_{ck} A_g + (0.67 f_y - 0.4 f_{ck}) \cdot A_{sc}]$$

$$3000 \times 10^3 = 1.05\left[\left(0.4 \times 25 \times \frac{\pi}{4} \times 450^2\right) + (0.65 \times 415 - 0.4 \times 25) \times A_{sc}\right]$$

Transverse reinforcement:

$$\phi_s \geq \text{Maximum} \begin{cases} \bullet\ \phi_{long,max}/4 = \frac{32}{4} = 8mm \\ \bullet\ 6\ mm \end{cases}$$

Assuming $\phi_s = 8mm$

$D_c = D - 2 \times$ clear cover

$= 450 - 2 \times 40$

$D_c = 370\ mm$

Now,

$$\frac{\text{Volume of spiral per pitch}}{\text{Volume of core per pitch}} \geq \frac{0.36 f_{ck}}{f_y}\left(\frac{A_g}{A_c} - 1\right)$$

$$\frac{\frac{\pi}{4} \times \phi_s^2 \times \pi (D_c - \phi_s)}{\frac{\pi}{4} \times D_c^2 \times \text{pitch}} \geq \frac{0.36 f_{ck}}{f_y}\left(\frac{\pi/4\ D^2}{\pi/4\ D_c^2} - 1\right)$$

$$\frac{8^2 \times \pi \times (370 - 8)}{370^2 \times \text{Pitch}} \geq \frac{0.36 \times 25}{415} \times \left(\frac{450^2}{370^2} - 1\right)$$

Pitch $\leq 51.17\ mm$

Dia should be such that this value must be more than 25mm

$$S/\text{pitch} \leq \text{Minimum} \begin{cases} \bullet\ 75\ mm \\ \bullet\ \frac{D_c}{6} = \frac{370}{6} = 61.66\ mm \end{cases}$$

$$S/\text{pitch} \geq \text{Maximum} \begin{cases} \bullet\ 25\ mm \\ \bullet\ 3\phi_s = 3 \times 8 = 24\ mm \end{cases}$$

Providing spiral of 8ϕ @ 50mm c/c

Ex. Design rectangular section of column subjected to factored axial load 4000kN. $L_{unsupported} = 3.4\,m$, pin connected at one end and fixed at another end, Fe415 and M20.

⇒ In general, 1-2 % of gross area is provided as longitudinal reinforcement. Considering 1.5 % as longitudinal R/F for this problem.

Section size is required to classify column as axially loaded short column. Since section size is not known so assuming column as axially loaded short column.

Step 1: Gross area:

$$P_u = 0.4 f_{ck} \cdot A_g + (0.67 f_y - 0.4 f_{ck}) \cdot A_{sc}$$

$$4000 \times 10^3 = 0.4 \times 20 \times A_g + (0.67 \times 415 - 0.4 \times 20) \times \frac{1.5}{100} \times A_g$$

$$\Rightarrow A_g = 331929.54\,mm$$

Assuming $D/b = 1.25$ (Preferably 1 to 3)

$$bD = A_g$$

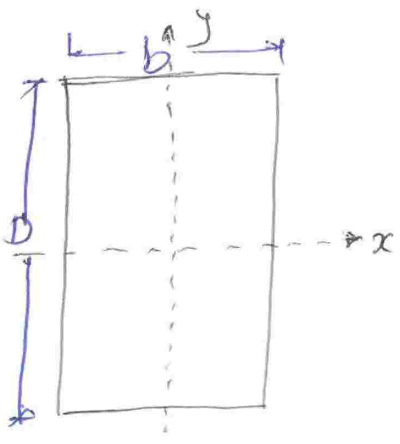
$$b \times 1.25 b = A_g$$

$$b = 515.31\,mm$$

Section size smaller than this value results into higher % of steel.

Assuming $b = 500\,mm$

$D = 1.25 b = 1.25 \times 500 = 625\,mm.$



Slenderness ratio :-

$$\lambda = \frac{L_{eff}}{b}$$

$$= \frac{0.8 \times 3.4 \times 10^3}{500}$$

$$\lambda = 5.44 < 12$$

short.

$$e_{min,x} = \text{Maximum} \begin{cases} \cdot \dfrac{L_{unsupported}}{500} + \dfrac{D}{30} \\ \cdot\ 20mm \end{cases}$$

$$= \text{Maximum} \begin{cases} \cdot \dfrac{3.4 \times 10^3}{500} + \dfrac{625}{30} = 27.63mm. \\ \cdot\ 20mm \end{cases}$$

$e_{min,x} = 27.63\ mm < 0.05D\ (31.25mm)$

$$e_{min,y} = \text{Maximum} \begin{cases} \cdot \dfrac{L_{unsupported}}{500} + \dfrac{b}{30} \\ \cdot\ 20\ mm \end{cases}$$

$$= \text{Maximum} \begin{cases} \cdot \dfrac{3.4 \times 10^3}{500} + \dfrac{500}{30} = 23.46 \\ \cdot\ 20\ mm \end{cases}$$

$e_{min,y} = 23.467 < 0.05b\ (25mm)$

So column is axially loaded.

Now,

$$P_u = 0.4 f_{ck} \cdot A_g + (0.67 f_y - 0.4 f_{ck}) \cdot A_{sc}$$

$$4000 \times 10^3 = 0.4 \times 20 \times 500 \times 625 + (0.67 \times 415 - 0.4 \times 20) \times A_{sc}$$

$\Rightarrow \quad A_{sc} = 5554.52\ mm^2$

Atleast 3-bars on each face (total 8-bars) are required to satisfy maximum spacing criteria. (300mm)

$\Rightarrow$ Providing $4 - 32\phi + 2 - 28\phi$

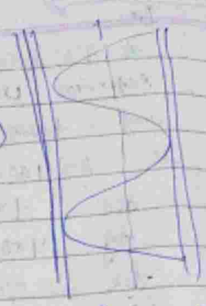
Transverse R/F

$$\phi \geq \text{Maximum} \begin{cases} \cdot\ \phi_{long,max}/4 = \dfrac{32}{4} = 8mm \\ \cdot\ 6mm \end{cases}$$

$$S/pitch \leq \text{Minimum} \begin{cases} \cdot \text{ Least lateral dimension} = 500mm \\ \cdot\ 16\,\phi_{long,min} = 16 \times 28 = 448mm. \end{cases}$$

For circular column :-

(spiral) Helical reinforcement



$$P = 1.05(0.4 f_{ck} A_c + 0.67 f_y A_{sc})$$

Ex:- Design a circular column of diameter 400mm subjected to a load of 1200kN. The column is having spiral ties. The column is 3m long and is effectively held in position at both ends but not restrained against rotation. Use M25 concrete & Fe415 steel.

Sol:- Given data :-

$L = 3m = 3000mm$

$P = 1200kN \Rightarrow P_u = 1.5 \times 1200 = 1800kN$

$D = 400mm$

from (M25 concrete & Fe415 steel) $f_{ck} = 25 N/mm^2$ & $f_y = 415 N/mm^2$

From IS code 456:2000, Table 11.1,

since the column effectively held in ~~pose~~ position at both ends but not restrained against rotation.,

effective length, $l_{eff} = 1.0L$

Slenderness ratio, $l/d = \frac{3000}{400} = 7.5 < 12$

∴ Hence it is a short column.

~~Min^m eccentricity~~

$$A_g = \frac{\pi}{4} \times 400^2 = 125663.7 \, mm^2$$

$$A_c = A_g - A_{sc} \doteq 125663.7 - A_{sc}$$

$$\text{Minimum eccentricity } (e_{min}) = \frac{L}{500} + \frac{D}{30}$$

$$= \frac{3000}{500} + \frac{400}{30} = 19.33 > 20mm$$

Take

$e_{min} = 20mm$ & $\frac{e_{min}}{D} = \frac{20}{400} = 0.05$

For column with spiral ties,

$P_u = 1.05\,(0.4 f_{ck} A_c + 0.67 f_y A_{sc})$

$\Rightarrow 1800\times10^3 = 1.05\,(0.4\times25\times(125663.7 - A_{sc}) + 0.67\times415\times A_{sc})$

$\Rightarrow 268.05\,A_{sc} = 480531.15$

$\Rightarrow A_{sc} = 1793\ mm^2$

Percentage of steel $= \dfrac{100\times A_{sc}}{A_g} = \dfrac{100\times1793}{125663.7} = 1.43\%$

it is bet? 0.8 and 4%. Hence ok.

Using 20 mm dia bars $A_\phi = 314\ mm^2$ $(\because \frac{\pi}{4}\times20^2 = 314)$

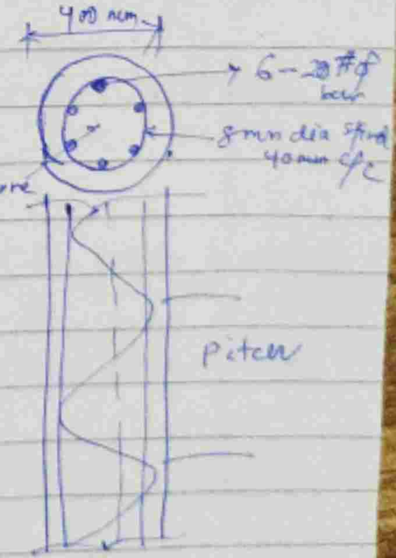
No of bars required $= \dfrac{1793}{314} = 4.7$ say 6 nos.

$\therefore$ Provide 6 – 20 mm dia bars.

A_{sc} provided $= 6\times\frac{\pi}{4}\times20^2 = 1884\ mm^2$.

Helical Reinforcement :-

Assuming Clear cover = 50 mm,



Core diameter = 400 – 2×50

= 300 mm

Area of core $= \frac{\pi}{4}\times300^2 - 1884 = 68801.8\ mm^2$

Assuming pitch = P

volume of core per pitch = 68801.8 × P

using 8 mm ϕ spiral.

Volume of one spiral per pitch

$= \frac{\pi}{4}\times8^2\times\pi\,(300-8)$

$= 46110.8\ mm^2$.

$\dfrac{\text{Volume of Helical reinforcement}}{\text{volume of core}} = \dfrac{46110.8}{68801.8\,P}$

As per IS code, $\dfrac{46110.8}{68801.8\,P} \geq 0.36\left(\dfrac{A_g}{A_c} - 1\right)\dfrac{f_{ck}}{f_y}$

Atleast 3 bars on each face (total 8 bars) are required

$$\leq 0.36\left(\frac{125663.7}{68801.8} - 1\right)\frac{25}{415}$$

$P \geq 37$ mm .

Max^m pitch should not be more than 75mm or

$\frac{\text{Core diameter}}{6} = \frac{300}{6} = 50$ mm .

Min^m Pitch :-

(i) 25mm .

ii) 3 x dia of helical Reinforcement = 3x8 = 24mm .

∴ Provide 8mm ϕ spirals @ 40mm c/c .

~~T - Beam~~ :-

11. Slab

11.1 Introduction:

Slab is a structural member of which one dimension is very small as compared to other two dimensions and primarilly subjected to bending.

11.2 Classification of Slab:

A) Based on Shape:

1. Rectangular
2. Circular
3. Triangular
4. Any other shape.

B) Based on Bending behaviour.

1. One-way slab.
2. Two-way slab.

C) Based on Type of Construction:

1. Slab/Solid slab.
2. Flat slab.
3. Slab with opening.
4. Waffled Slab.

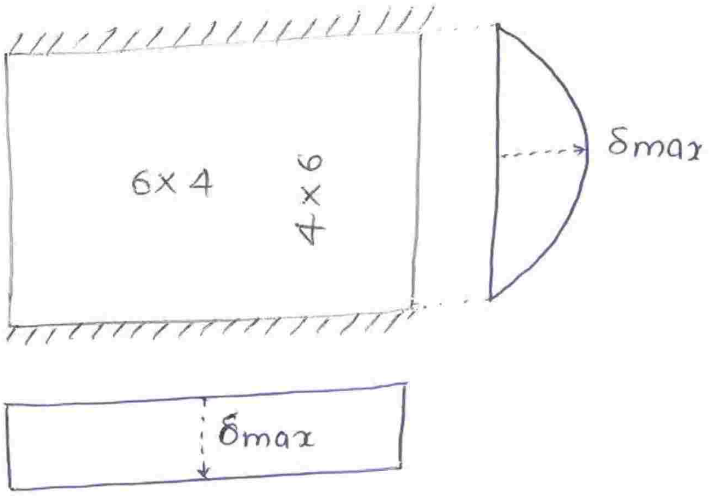
D) Based on Type of Loading:

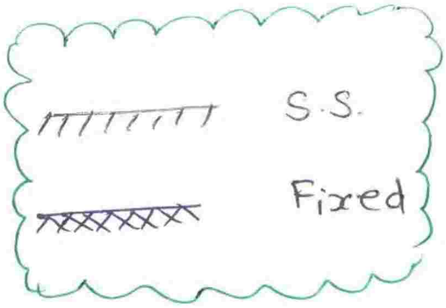
1. Subjected to point load. (Bridge slab)
2. Subjected to UDL

11.3.1 One-way Slab:

If bending in one-direction is very significant than other orthogonal direction then slab is classified as one-way.

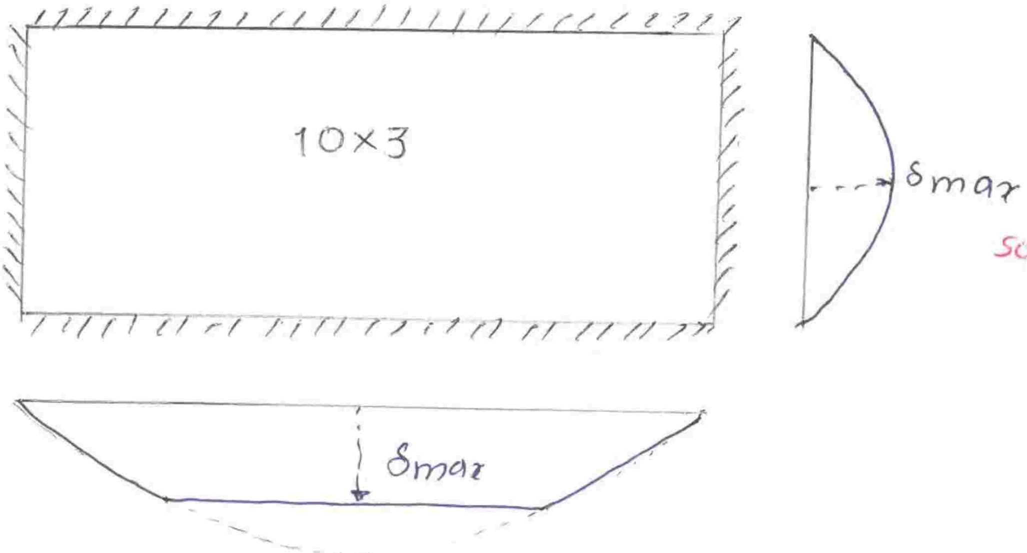
A) Rectangular slab supported from opposite edges:





Rectangular slab supported from opposite edges is always one-way, irrespective of dimension

B) Rectangular slab supported from all four edges.



Aspect ratio $= \frac{10}{3} = 3.33 > 2$

So, one way

A rectangular slab supported from all 4-sides is considered as one way if aspect ratio is more than 2.

$$\text{aspect ratio} = \frac{\text{Longer effective span}}{\text{Shorter effective span}}$$

$$= \frac{L_y}{L_x} > 2$$

* **Note:**

Aspect ratio is valid only for rectangular slab supported from all four sides.

11.3.2 Two-way Slab:

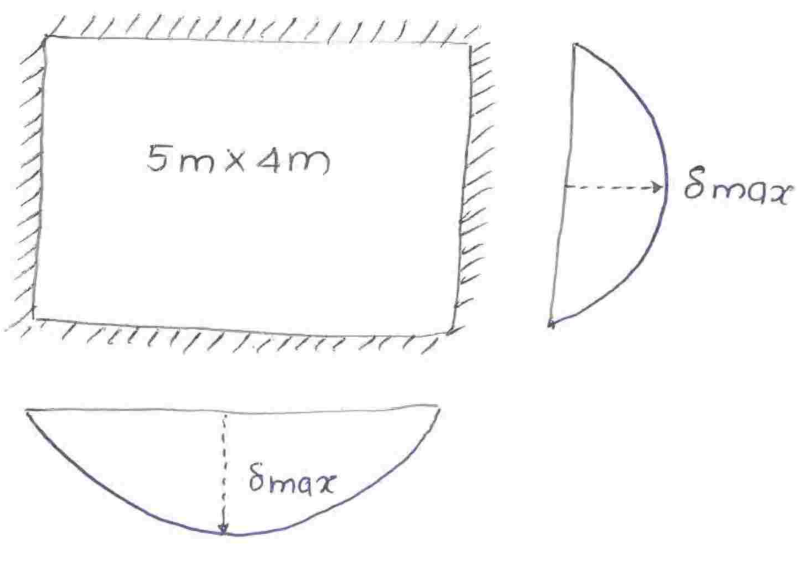
If bending is comparable in two-orthogonal directio[n] then such slabs are called as two-way slabs.

A rectangular slab is classified as two-way based on following two-conditions.

1) Aspect Ratio:

2) Supporting condition.

If aspect ratio ≤ 2 and rectangular slab is supported from all 4-sides. then only rectangular slabs are classified as two-way.



$$\text{Aspect ratio} = \frac{L_y}{L_x} = \frac{5}{4} = 1.25 < 2$$

So, Two-way

11.5.3 Deflection Control:

If rectangular two way slab is supported from all 4-sides with shorter span upto 3.5m and loading upto 3 kN/m² then $\frac{L_{eff}}{D}$ ratio should be less than values given below.

Supporting condition	Fe 250	Fe 415
• Simply supported	35	$0.8 \times 35 = 28$
• Continuous	40	$0.8 \times 40 = 32$

* Note:

• Fe 250

$\frac{L_{eff}}{D} < 35$

$\Rightarrow D > \frac{L_{eff}}{35}$

• Fe 415

$\frac{L_{eff}}{D} < 28$

$\Rightarrow D > \frac{L_{eff}}{28}$

One way slab is designed by considering a strip of unit width.

Step 1: Assume suitable value of 'd' for preliminary design.

$$\frac{l_{eff}}{d} < K_1 K_2 K_3 K_4 \text{ (value)}$$

K_1 = depends on span

$K_2 = 1.25$ (corresponding to generally provided % of tension R/F)

$K_3 = 1$ (Singly R/F)

$K_4 = 1$ (Rectangular)

Calculate D by assuming suitable value of effective cover.

Step 2: Calculate Effective span.

Step 3: Calculate DL and design BM. Use BM co-efficient for continuous slab.

Step 4: Calculate 'd' required for balanced section.

$$BM_u = M_{u,lim}$$

$$\Rightarrow BM_u = Q b d^2$$

$$\Rightarrow d = ??$$

where, $b = 1000$ mm

d calculated here should be less than 'd' assumed in Step 1. otherwise d suitably higher than calculated here and repeat Step 2 to Step 4.

Step 5: Since, 'd' provided is greater than d required in Step 4 so section is under reinforced.

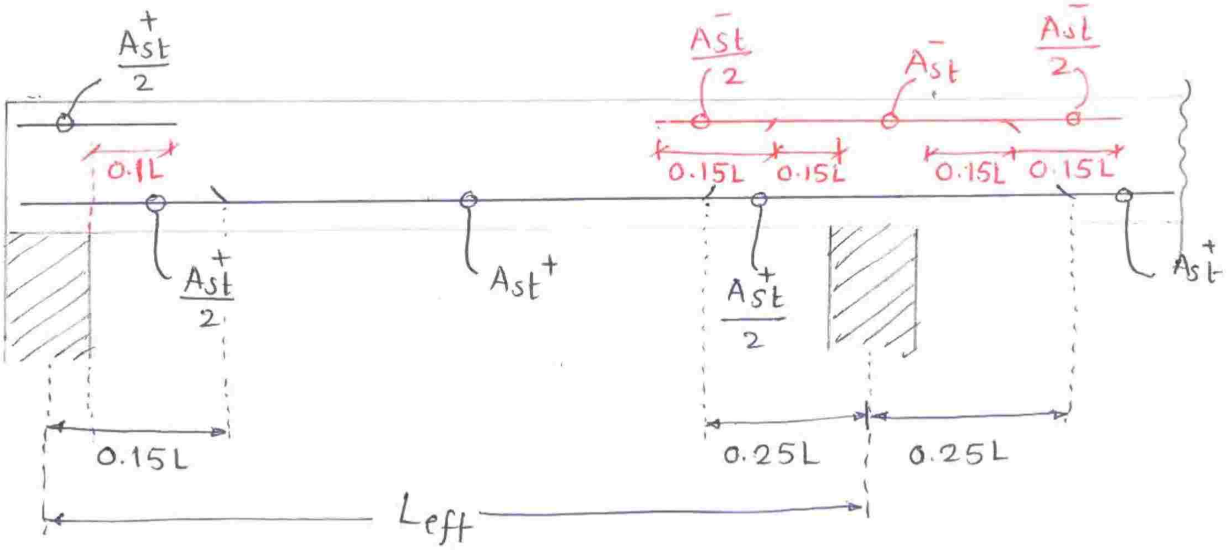
$$A_{st} = \frac{0.5 f_{ck} b d}{f_y}\left[1 - \sqrt{1 - \frac{4.6 BM_u}{f_{ck} b d^2}}\right]$$

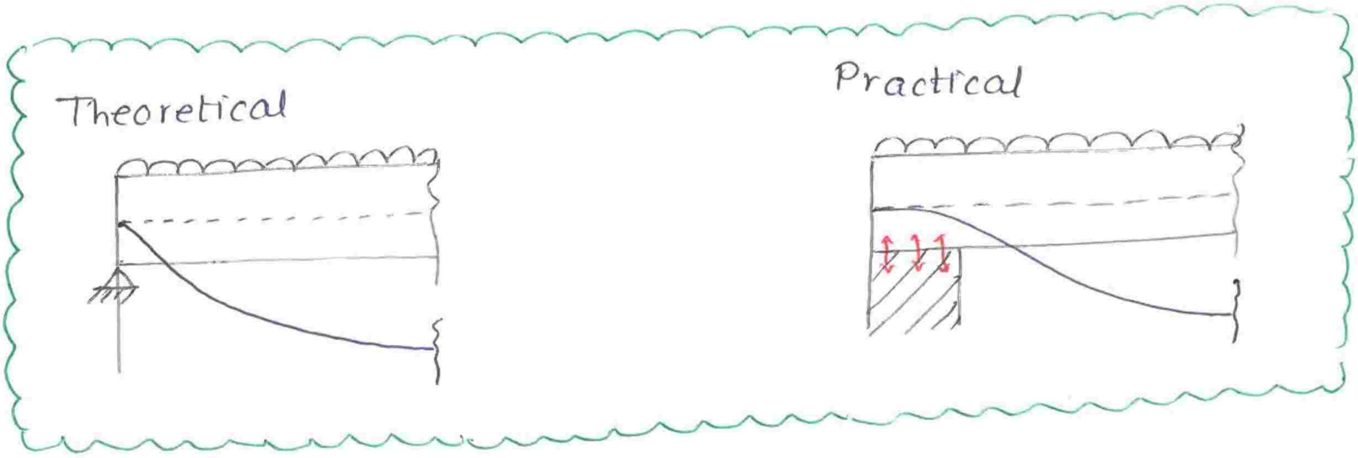
where, $b = 1000$ mm

Step 6: Ast calculated above should not be less than Ast,min

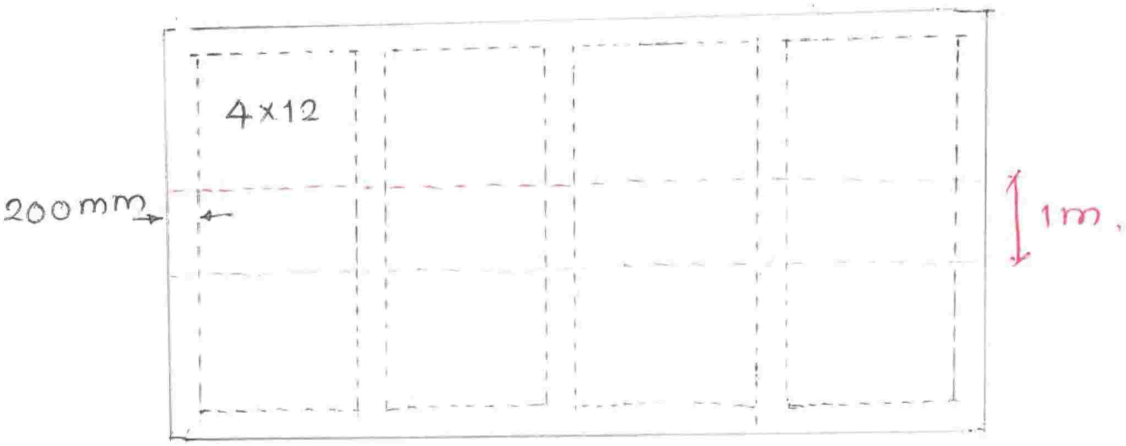
Step 7: Provide distribution reinforcement

Step 8: Reinforcement detailing.





Ex. Design the slab given below. LL is $6 kN/m^2$, FF 65mm thick, unit weight of floor F. (FF) $22 kN/m^3$, M20, Fe415, mild exposure, slab is continuous over 4-equal spans.



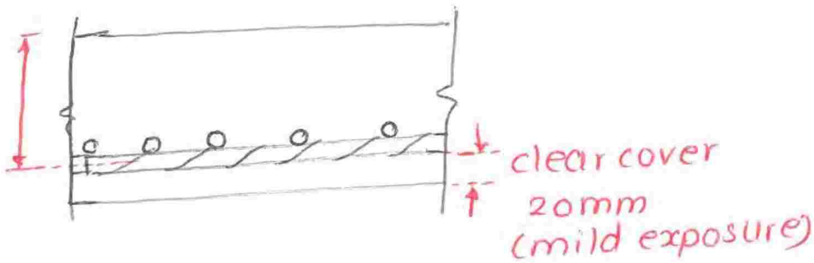
Step 1: Depth:

$$\frac{L_{eff}}{d} < K_1 K_2 K_3 K_4 \text{ (value)}$$

$$\frac{4.2\times10^3}{d} < (1)\ (1.25)(1)(1)\ \left(\frac{20+26}{2}\right)$$

$$d > 146.08\text{mm}$$

Assuming $d = 150$mm



$D = d$ + allowance for R/F + clear cover

$= 150 + \underset{\text{(assumed)}}{10} + \underset{\text{(mild exposure)}}{20}$

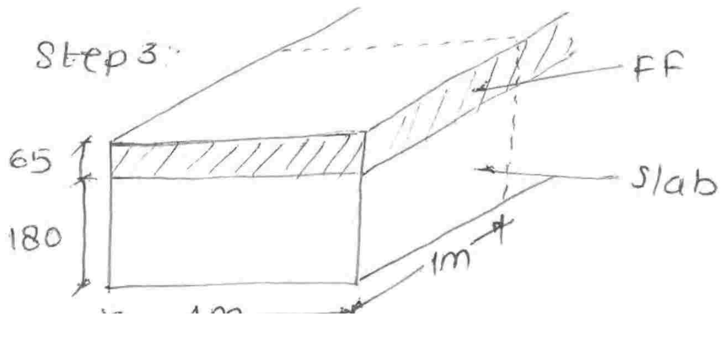
$D = 180$mm

Step 2: IS 456 doesn't provide L_{eff} for all types of supporting conditions. Considering simply supported as a ~~nearest~~ similar supporting condition for the present case

$$L_{eff} = \text{Minimum} \begin{cases} \cdot\ L_c + d = 4 + 0.15 = 4.15\text{m} \\ \cdot\ \frac{b_1}{2} + L_c + \frac{b_2}{2} = \frac{0.2}{2} + 4 + \frac{0.2}{2} = 4.2\text{m} \end{cases}$$

$L_{eff} = 4.15$ m

Step 3:



Self weight $= 0.18\times1\times1\times25$

$= 4.5$ kN/m^2

FF. $= 0.065\times22 = 1.93$ kN/m^2

Fixed Load $= 5.93$ kN/m^2

Factored fixed load $= 1.5 \times (4 + 1.93) = 8.89\ kN/m^2$

Factored Non-fixed load $= 1.5 \times 6 = 9\ kN/m^2$

$$BM_u^+ = +\frac{1}{12} w_F L_{eff}^2 + \frac{1}{10} w_{NF} L_{eff}^2$$

$$= \frac{1}{12} \times 8.89 \times (4.15)^2 + \frac{1}{10} \times 9 \times (4.15)^2$$

$BM_u^+ = 28.26\ kNm$ (Sagging)

$$BM_u^- = -\frac{1}{10} w_F L_{eff}^2 - \frac{1}{9} w_{NF} L_{eff}^2$$

$$= -\frac{1}{10} \times 8.89 \times (4.15)^2 - \frac{1}{9} \times 9 \times (4.15)^2$$

$= -32.53\ kNm$

$BM_u^- = 32.53\ kNm$ (Hogging)

Step 4: 'd' required for balanced section.

$BM_u^- = M_{u,lim}$

$BM_u^- = 0.138 f_{ck} b d^2$

$32.53 \times 10^6 = 0.138 \times 20 \times 1000 \times d^2$

$d = 108.56\ mm < 150\ mm$ (OK)

Step 5: Ast required

$$A_{st}^+ = \frac{0.5 f_{ck} b d}{f_y}\left[1 - \sqrt{1 - \frac{4.6\, BM_u^+}{f_{ck} b d^2}}\right]$$

$$= \frac{0.5 \times 20 \times 1000 \times 150}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 28.26 \times 10^6}{20 \times 1000 \times 150^2}}\right]$$

$A_{st}^+ = 566.46\ mm^2$

Similarly

$A_{st}^- = 661.48\ mm^2$

Step 6: $A_{st,min} = 0.12\% \, bD$

$$= 0.12 \times \frac{1}{100} \times 1000 \times 180$$

$A_{st,min} = 216 mm^2$

Difference between A_{st}^{+} and A_{st}^{-} is not very significant so providing A_{st}^{-} for both.

$A_{st}^{-} = 661.48 mm^2$

Assuming $\phi = 10 mm$

$$\text{Spacing} = \frac{1000}{\text{No. of bar}}$$

$$= \frac{1000}{A_{st}^{-} / \frac{\pi}{4}\phi^2}$$

$$= \frac{1000}{661.48 / \frac{\pi}{4} \times 10^2} = 118.73 \, mm$$

Providing 10ϕ @ 100

Step 7: Distribution Steel

$A_{st,min} = 0.12\% \, bD = 216 mm^2$

Assuming $\phi = 8 mm$

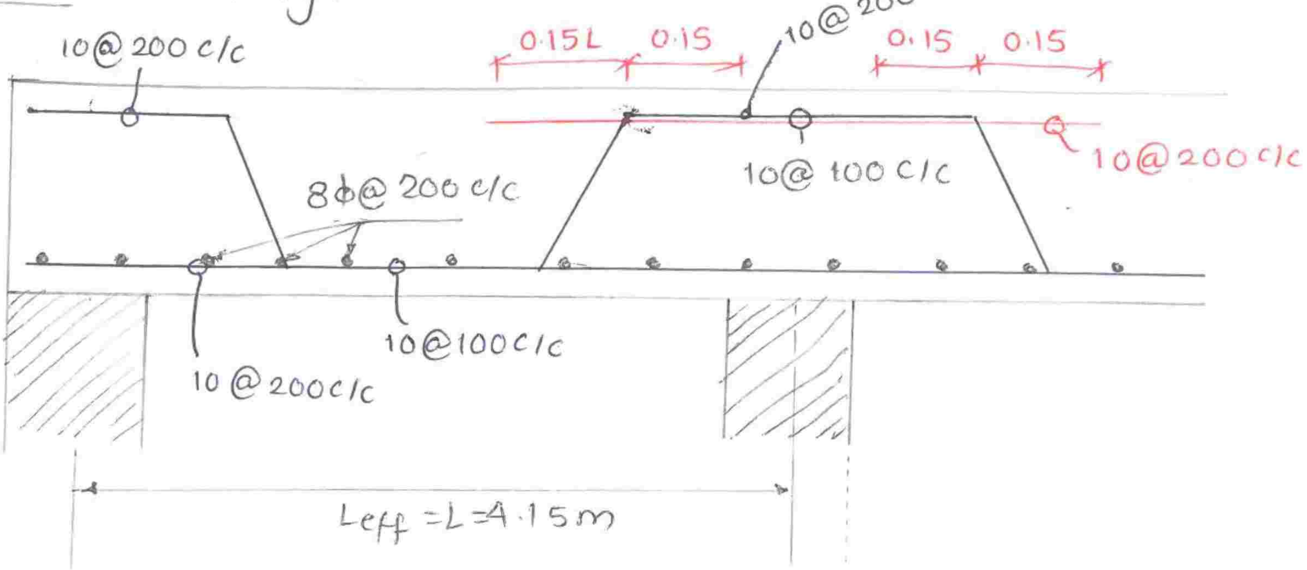
$$\text{Spacing} = \frac{1000}{\text{No. of bar}}$$

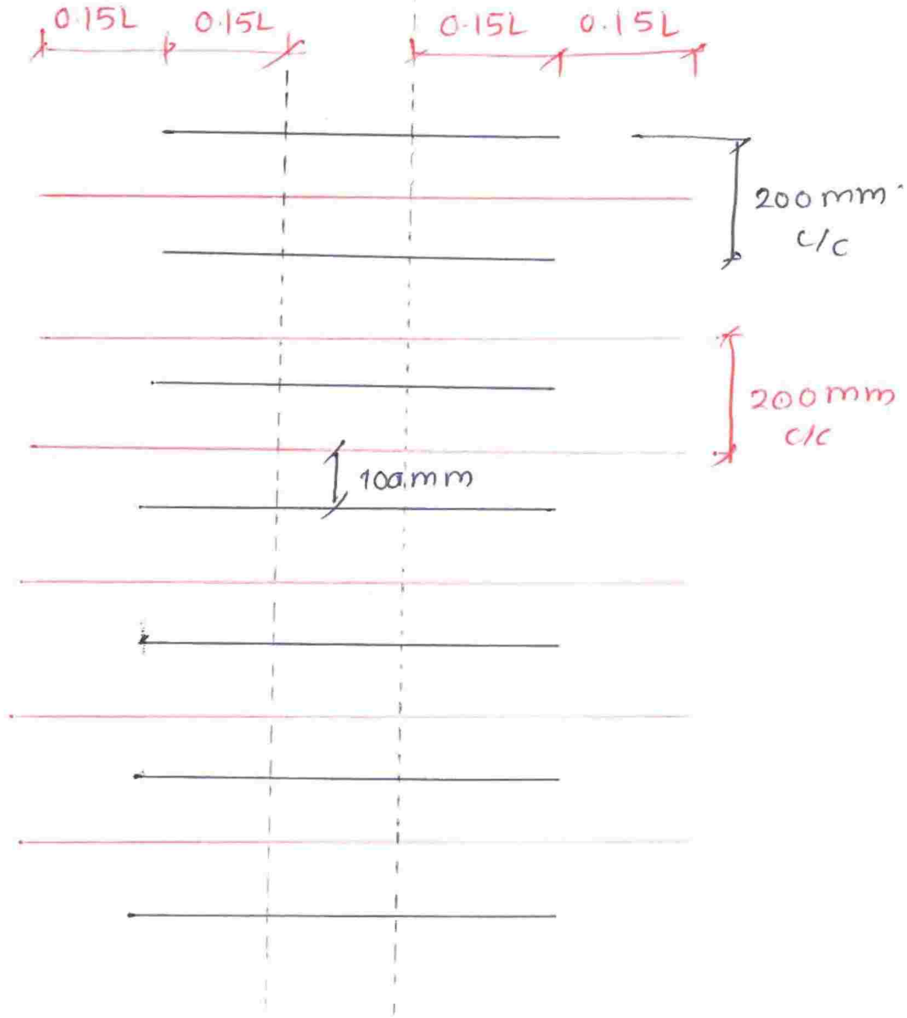
$$= \frac{1000}{A_{st,min} / \frac{\pi}{4}\phi^2}$$

$$= \frac{1000}{216 / \frac{\pi}{4} \times 8^2} = 232.71 \, mm$$

Providing 8ϕ @ 200 mm c/c

Step 6: Detailing,





12. Foundation

12.1 Introduction:

Foundation is a structural element below ground level that transfers load of superstructure to the soil safely.

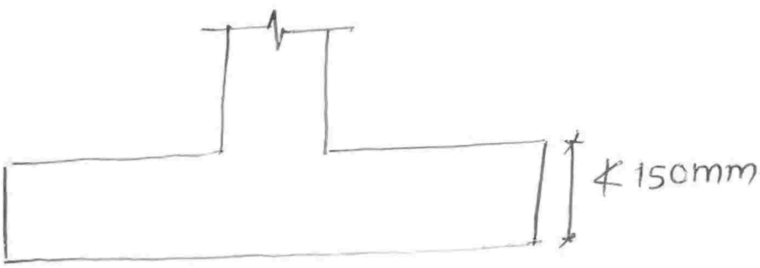
Foundation

- Shallow (footing)
  - Isolated
  - Combined
  - Strap
  - Raft/Mat
- Deep
  - Pile
  - Well

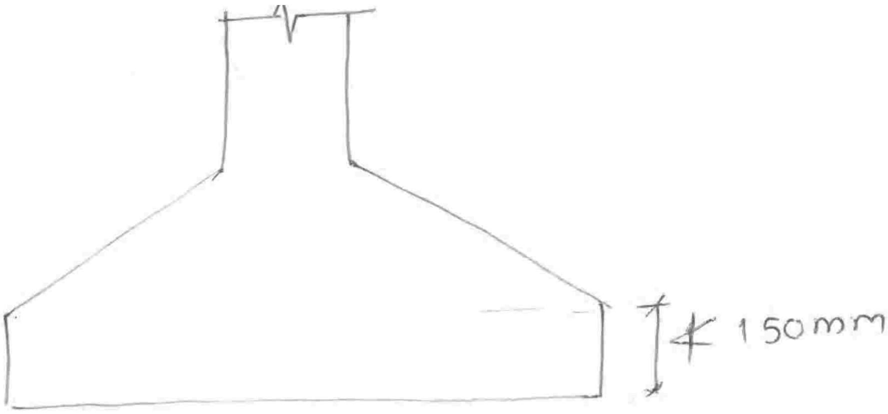
12.2 Description of Different types of footing.

12.2.1 Isolated Footing:

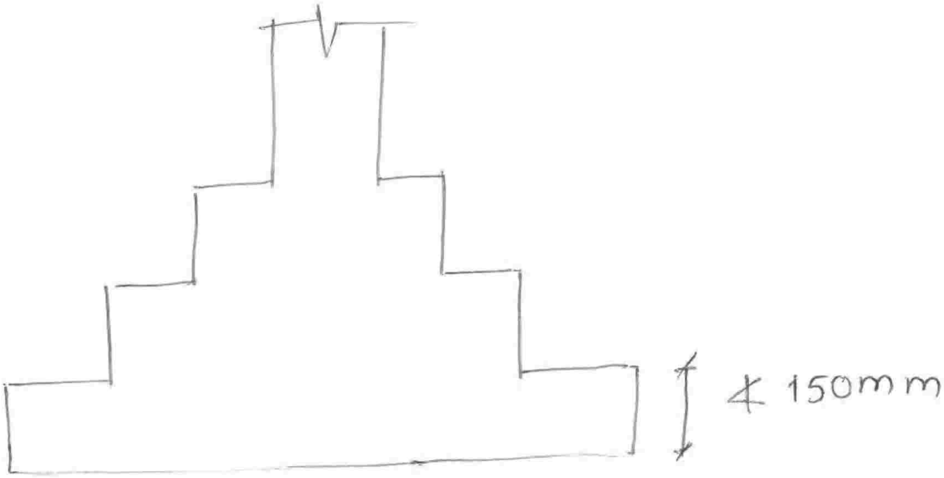
If one footing is for one column/wall then it is termed as Isolated footing. It may be square, rectangular, circular in plan and of uniform thickness, stepped or sloped in elevation.



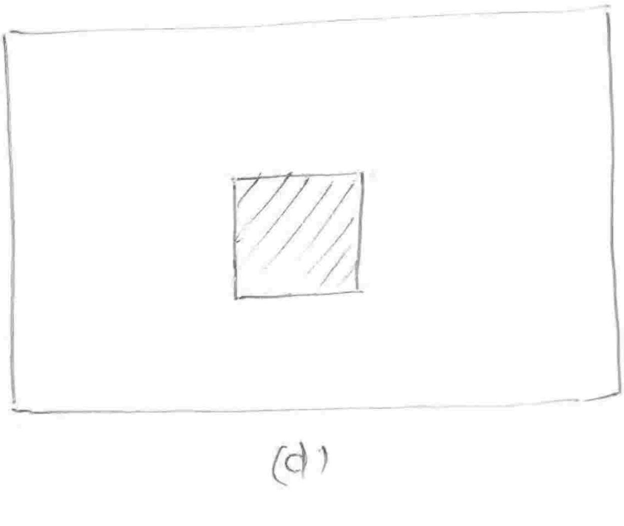
(a)



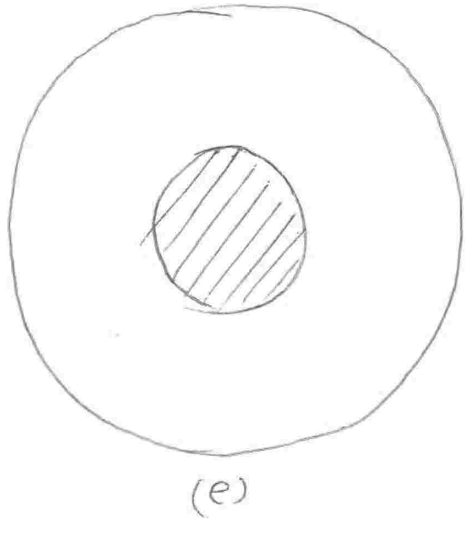
(b)



(c)



(d)

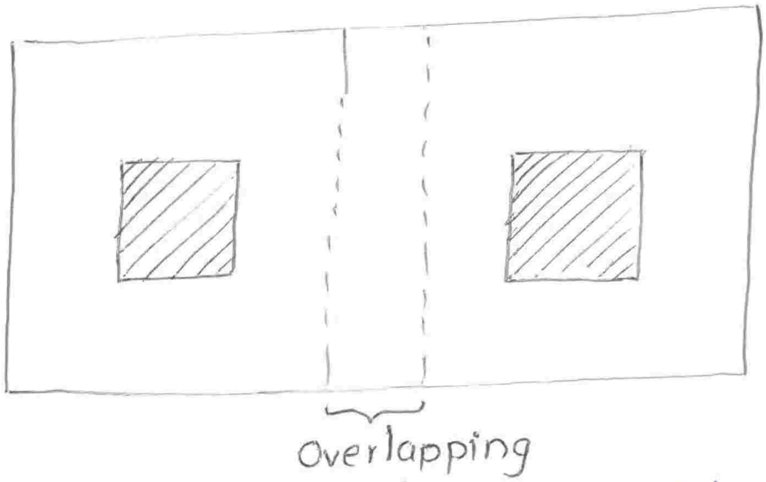


(e)

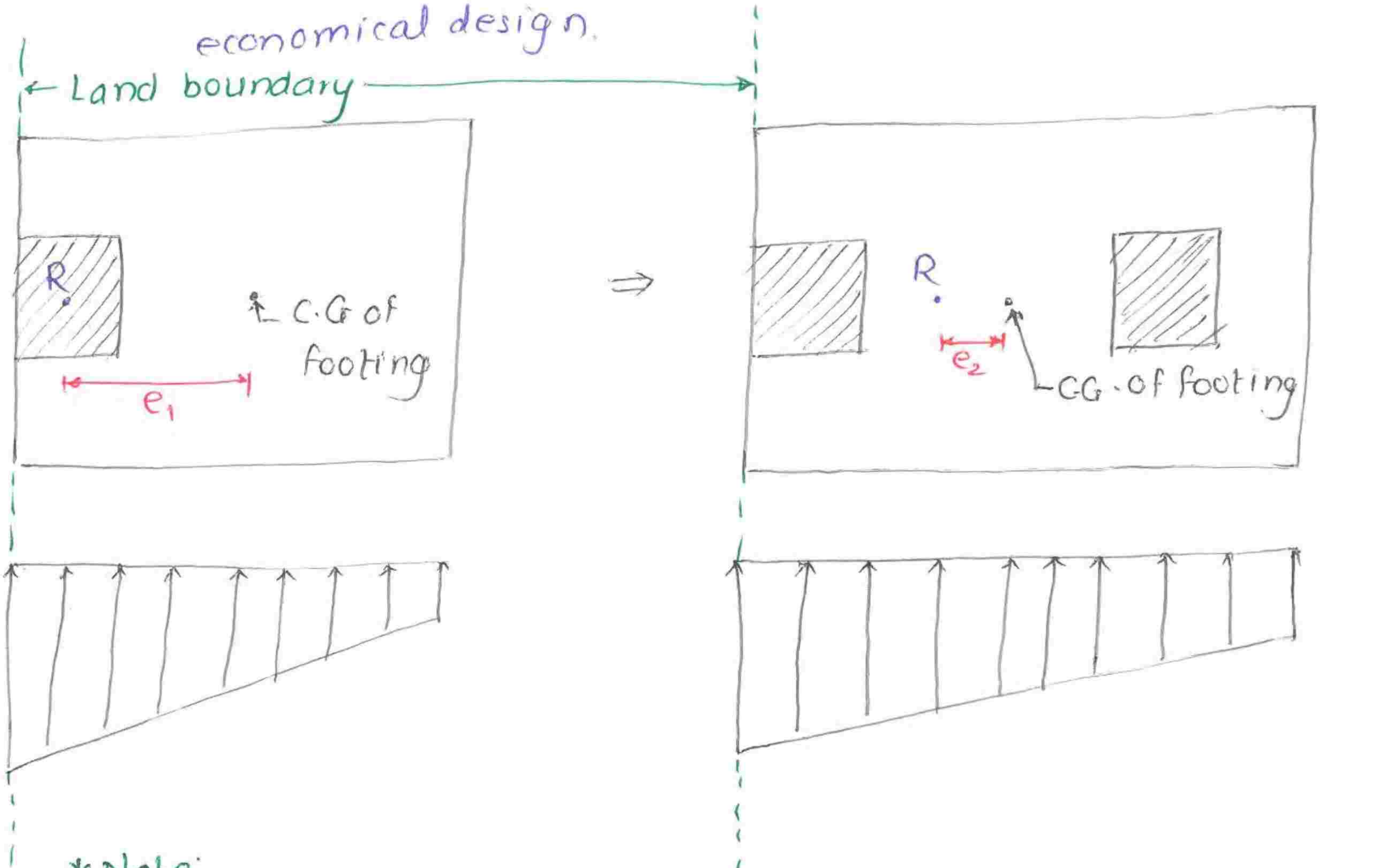
12.2.2 Combined Footing

It is provided in following two cases:

Case I: If columns are closely spaced and their isolated footings are overlapping then combined footing is preferable.



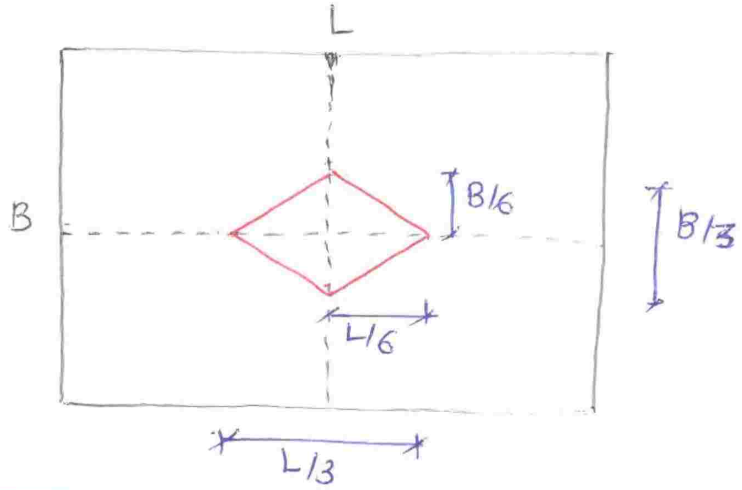
Case II: If column is placed at land boundary then its isolated footing is combined with isolated footing of other column to get desired pressure distribution for economical design.



* Note:

- For uniform pressure on soil below footing, resultant of all loads must pass through C.G. of plan area of footing.

For non-zero upward pressure from soil (no lifting condⁿ) resultant must pass through middle third of plan area. This is also called as Middle Third Rule.



Ex. Calculate maximum and minimum pressure exerted by soil on footing base of plan area (3 x 4) m. This footing is supporting axial load 2000 kN and moment 400 kN·m about shorter side, at its base.

⇒

$$\sigma_{max/min} = \frac{P}{A} \pm \frac{M}{I} \cdot \frac{L}{2}$$

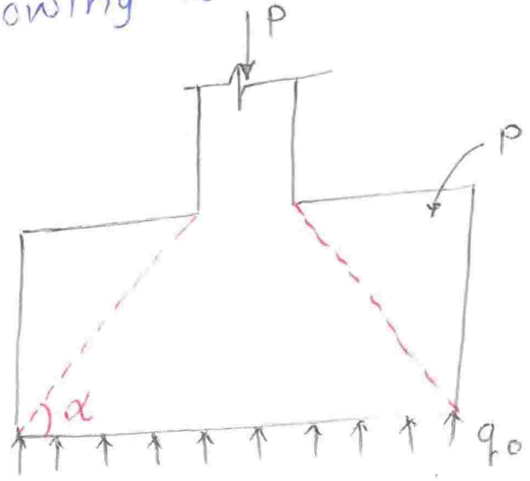
$$= \frac{2000}{3 \times 4} \pm \frac{400}{\frac{3 \times 4^3}{12}} \times \frac{4}{2}$$

$$\sigma_{max} = 216.67 \ kN/m^2$$

$$\sigma_{min} = 116.67 \ kN/m^2$$

12.4 Codal Provisions.

- Minimum slab thickness is 150mm.
- Minimum nominal cover 50mm
- Dimension of footing of PCC should be such that following condition must be satisfied,

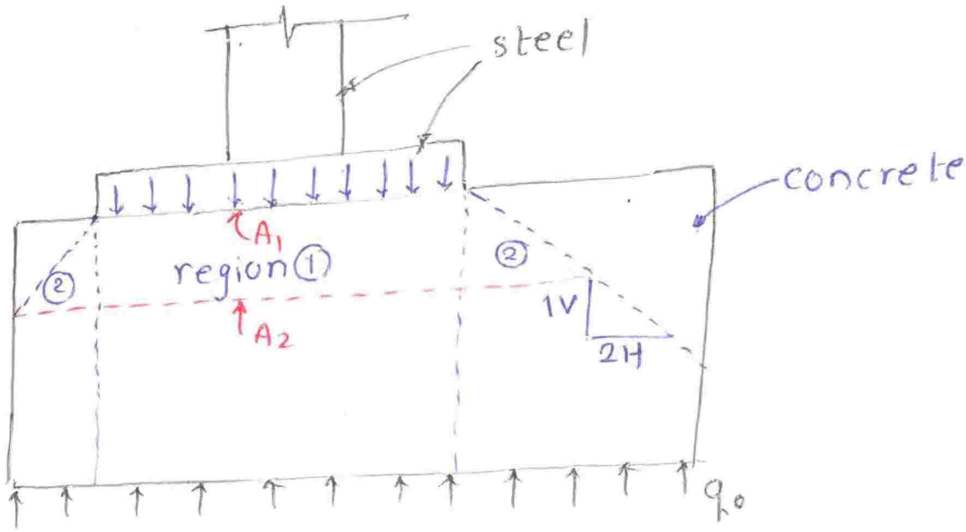


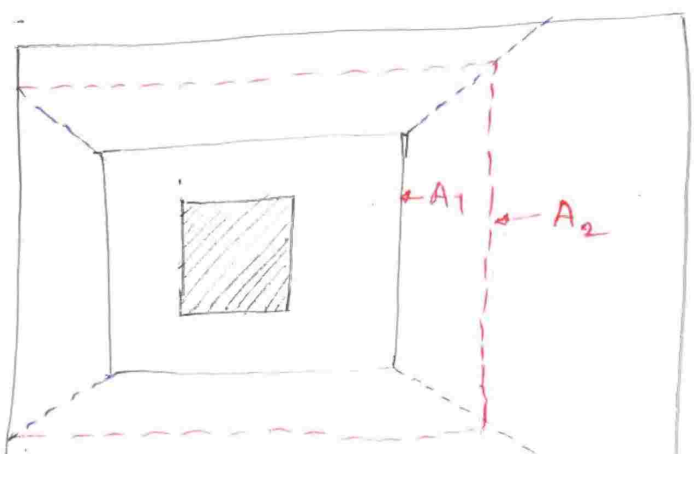
$$\tan\alpha \nless 0.9\sqrt{\frac{100q_o}{f_{ck}}+1}$$

- Maximum bearing strength of concrete is

$$0.45f_{ck}\sqrt{\frac{A_2}{A_1}}$$

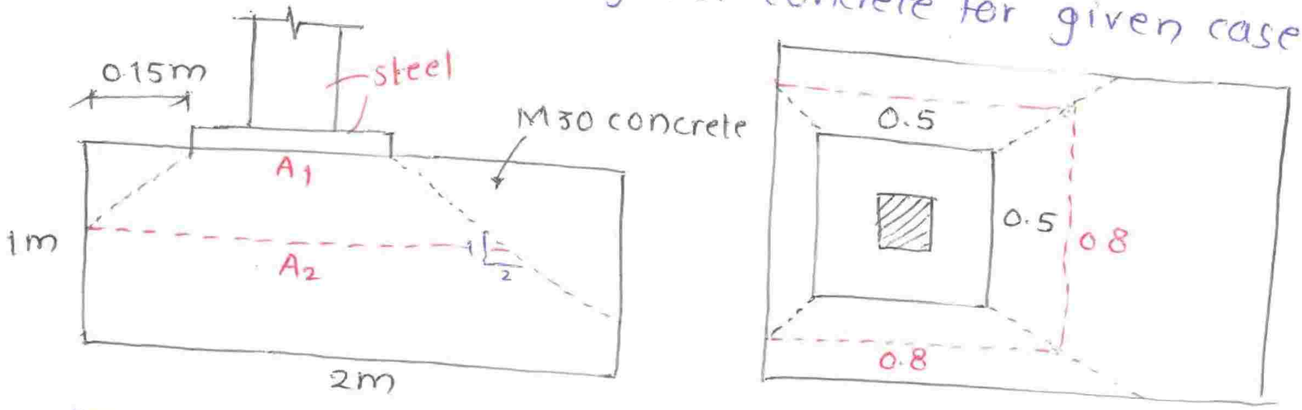
where, $\sqrt{\frac{A_2}{A_1}} \ngtr 2$





...g strength of concrete is enhanced by $\sqrt{\frac{A_2}{A_1}}$ because concrete of region ① is confined by concrete of region ②.

Ex. Calculate bearing strength of concrete for given case.



$$\Rightarrow \quad \sqrt{\frac{A_2}{A_1}} = \sqrt{\frac{0.8 \times 0.8}{0.5 \times 0.5}} = 1.6$$

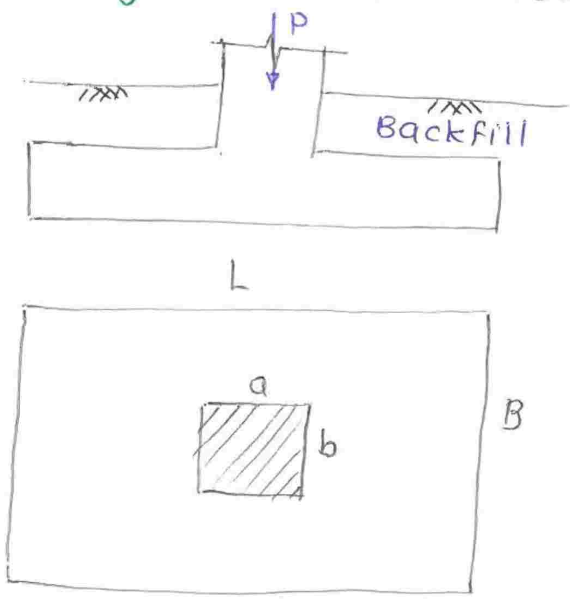
$$f_{br} = 0.45 f_{ck} \sqrt{\frac{A_2}{A_1}} = 0.45 \times 30 \times 1.6$$

$$\Rightarrow \quad f_{br} = 21.6 \text{ N/mm}^2$$

12.5 Design of Rectangular Isolated footing of uniform thickness subjected to Axial Load:

L>B

a>b



Step1: Take working axial load

Step2: Take safe bearing capacity (SBC) of soil

Step3: Take self weight of footing and backfill as 10% of axial working load. (based on experience)

Step4: Calculate plan area required for footing base slab.

$$A_{req} = \frac{P + 0.1P}{SBC}$$

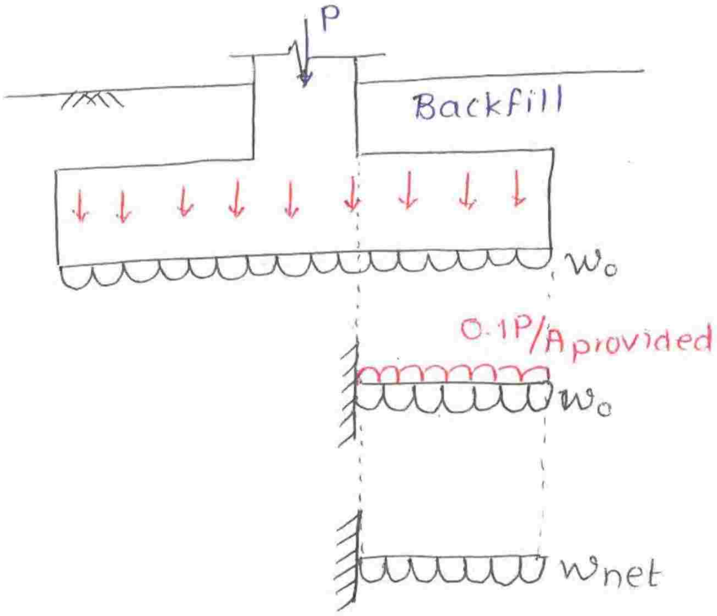
Step5: Provide dimension of footing in such a way that overhang on both sides of column should be approximately equal.

$$A_{provided} \geq A_{required}$$

Step 6: Calculate upward soil pressure on base slab.

$$w_o = \frac{P + 0.1P}{A_{provided}} \leq SBC$$

Step7: Calculate net upward pressure for design of base slab.



$$w_{net} = w_o - \frac{0.1P}{A_{provided}}$$

$$= \frac{P + 0.1P}{A_{prov.}} - \frac{0.1P}{A_{prov.}}$$

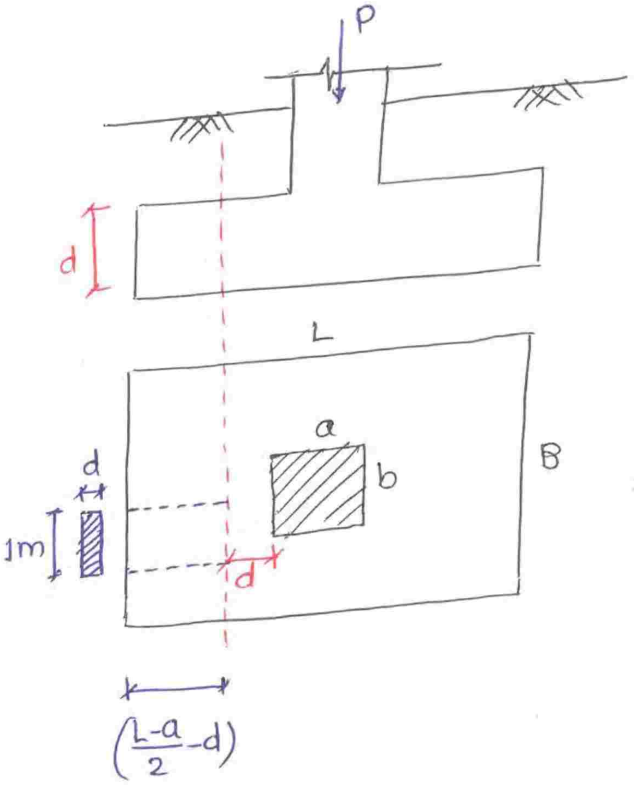
$$w_{net} = \frac{P}{A_{provided}}$$

Step8: Calculate factored Net upward pressure.

$$w_u = 1.5\, w_{net}$$

Step 9: Design for one way shear

Critical section for one way shear is at a distance 'd' from face of column/wall.



$$\text{Oneway shear stress} \le k\tau_c$$

$$\Rightarrow \frac{\text{Load on strip}}{\text{Resisting area}} \le k\tau_c$$

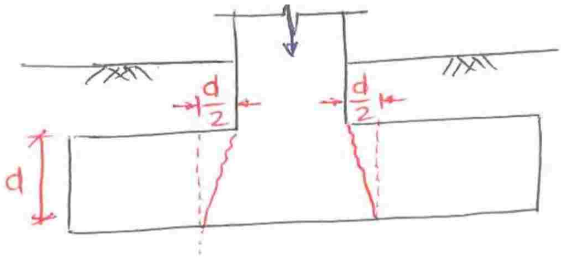
$$\Rightarrow \frac{w_u\left[\left(\frac{L-a}{2}-d\right)\times 1\right]}{1\times d} \le k\tau_c$$

Above expression is used to calculate 'd' required to prevent oneway shear failure.

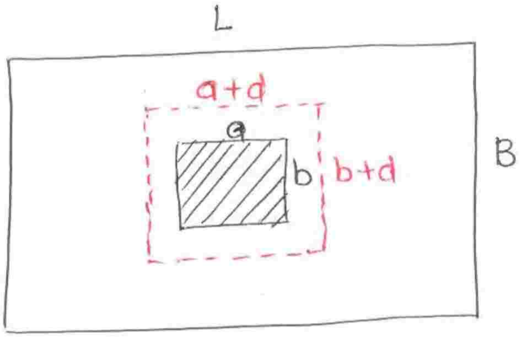
$\ast$ Note:- For preliminary design, considering $k=1$ and τ_c corresponding to 0.2% of longitudinal tension reinforcement.

Step 10: Design for punching / two way shear:

Critical section for punching shear is at a distance $d/2$ from face of column.



2-way shear stress $\leq k_\beta \, 0.25\sqrt{f_{ck}}$

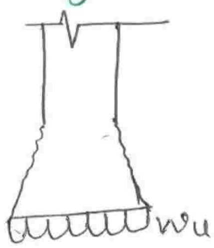


$$\Rightarrow \frac{\text{Punching SF}}{\text{Resisting area}} \leq k_\beta \, 0.25\sqrt{f_{ck}}$$

$$\Rightarrow \frac{P_u - W_u\left[(a+d)(b+d)\right]}{2\left[(a+d)+(b+d)\right] d} \leq k_\beta \, 0.25\sqrt{f_{ck}}$$

where, $k_\beta = \text{Minimum} \begin{cases} \bullet\ 0.5 + b/a \\ \bullet\ 1 \end{cases}$

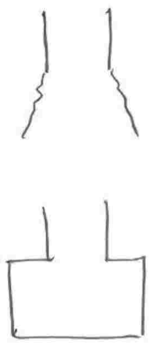
Punching SF :



Punching SF $= P_u - W_u\left[(a+d)(b+d)\right]$

where,

$P_u = 1.5P$



$*$ Note:

If footing fails in punching shear then depth is sufficiently increased and only step 10 is repeated.

Ex. Design an isolated footing of uniform thickness for column of section sizer (300 × 500) mm. Column is subjected to factored axial load 1800 kN. SBC = 130 kN/m², M25, Fe415, effective cover 75 mm.

⇒

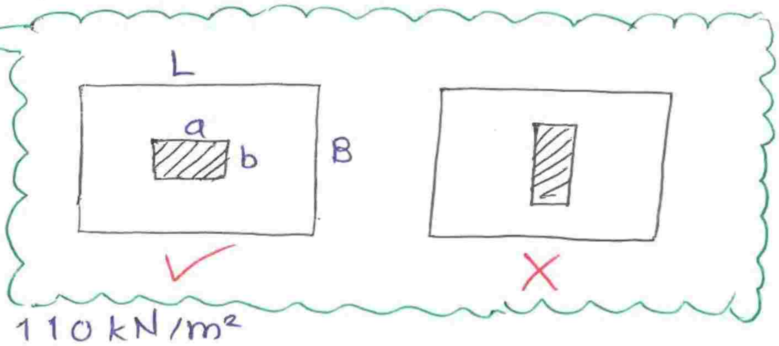
Step 1: $P = \frac{P_u}{1.5} = \frac{1800}{1.5} = 1200\,kN$

Step 2: $SBC = 130\,kN/m^2$

Step 3: Self wt. $= 0.1P = 120\,kN$

Step 4: $A_{req} = \frac{P + 0.1P}{SBC} = \frac{1200 + 120}{130} = 10.15\,m^2$

Step 5: Providing 3 × 4 m



Step 6: $w_o = \frac{P + 0.1P}{A_{provided}}$

$= \frac{1200 + 120}{12} = 110\,kN/m^2$

Step 7: $w_{net} = \frac{P}{A_{provided}} = \frac{1200}{3 \times 4} = 100\,kN/m^2$

Step 8: $w_u = 1.5\,w_{net} = 150\,kN/m^2$

Step 9: One way shear

$\tau_c = 0.32\,N/mm^2$

$= 320\,kN/m^2$

Now, One way shear stress $\leq k\,\tau_c$

$$\frac{w_u\left[\left(\frac{L-a}{2} - d\right) \times 1\right]}{1 \times d} \leq k\,\tau_c$$

$$\frac{150\left[\left(\frac{4-0.5}{2} - d\right) \times 1\right]}{1 \times d} \leq 1 \times 320$$

$d \geq 0.558\,m$

$d \geq 558\,mm$

$D = d + \text{effective cover} = 600 + 75$

providing d= 600mm $D = 675\,mm$

step 10: Two-way shear:

$$k_\beta = \text{Minimum} \begin{cases} \bullet\ 0.5 + b/a = 0.5 + 0.3/0.5 = 1.1 \\ \bullet\ 1 \end{cases}$$

$k_\beta = 1$

$0.25\sqrt{f_{ck}} = 0.25\sqrt{25} = 1.25\,N/mm^2 = 1250\,kN/m^2$

Two-way shear stress $\leq k_\beta\, 0.25\sqrt{f_{ck}}$

$$\frac{P_u - W_u\left[(a+d)(b+d)\right]}{2\left[(a+d)+(b+d)\right]\cdot d} \leq k_\beta\, 0.25\sqrt{f_{ck}}$$

$$\Rightarrow \frac{1800 - 150\left[(0.5+0.6)(0.3+0.6)\right]}{2\left[(0.5+0.6)+(0.3+0.6)\right]\times 0.6} \leq 1 \times 1250$$

$\Rightarrow 688.12 < 1250$ OK

Step 11: Design for Bending:

• Longer Overhang:

$$BM_{max} = \frac{w_u\left(\frac{L-a}{2}\right)^2}{2}$$

$$= \frac{150 \times \left(\frac{4-0.5}{2}\right)^2}{2}$$

$BM_{max} = 229.68\ kNm$

$M_{u,lim} = 0.138\, f_{ck}\, bd^2$

$= 0.138 \times 25 \times 1000 \times 600^2$ =>Mu, lim=1242knm

$$A_{st} = \frac{0.5 f_{ck} b' d}{f_y}\left[1 - \sqrt{1 - \frac{4.6 BM_{max}}{f_{ck} b' d^2}}\right]$$

$$= \frac{0.5 \times 25 \times 1000 \times 600}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 229.68 \times 10^6}{25 \times 1000 \times 600^2}}\right]$$

$A_{st} = 1093.87 \, mm^2$

Now,

$A_{st,min} = 0.12\% \; b'D$

$$= 0.12 \times \frac{1}{100} \times 1000 \times 675$$

$A_{st,min} = 810 \, mm^2$

$A_{st} = 1093.87 \, mm^2$

Assuming $\phi = 16 \, mm$

$$\text{Spacing} = \frac{1000}{\text{No. of bars}}$$

$$= \frac{1000}{A_{st} / \frac{\pi}{4}\phi^2}$$

$$= \frac{1000}{1093.87 / \frac{\pi}{4} \times 16^2}$$

Spacing $= 183.80 \, mm$

Providing 16ϕ @ $175 \, mm$ c/c.

- Shorter Overhang:

$$BM_{max} = \frac{w_u \left(\frac{B-b}{2}\right)^2}{2}$$

$$= \frac{150 \times \left(\frac{3-0.3}{2}\right)^2}{2}$$

$BM_{max} = 136.68 \, kNm.$

$M_{u,lim} = 1242 \, kN \cdot m$

$$A_{st} = \frac{0.5 f_{ck} b' d}{f_y}\left[1 - \sqrt{1 - \frac{4.6\ BM_{max}}{f_{ck} b' d^2}}\right]$$

$$= \frac{0.5 \times 25 \times 1000 \times 600}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 136.68 \times 10^6}{25 \times 1000 \times 600^2}}\right]$$

$A_{st} = 642.78\ mm^2 < A_{st,min}\ (810\ mm^2)$

So providing $A_{st,min}$

$A_{st} = 810\ mm^2$

Assuming $\phi = 12\ mm$

$$\text{Spacing} = \frac{1000}{\text{No. of Bars}} = \frac{1000}{A_{st} / \frac{\pi}{4} \times \phi^2}$$

$$= \frac{1000}{810 / \frac{\pi}{4} \times 12^2}$$

Spacing $= 139.63\ mm$

Providing 12ϕ @ 125 mm c/c

Step 12: Reinforcement Detailing:

$n_T =$ (No. of bars per meter × distance) + 1

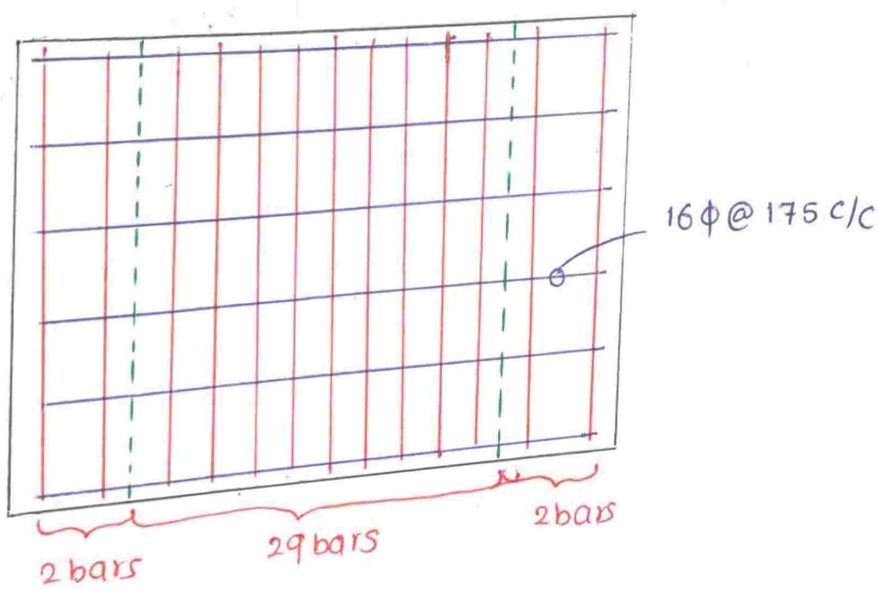
$$= \left(\frac{1000}{\text{spacing}} \times \text{distance}\right) + 1$$

$$= \left(\frac{1000}{125} \times 4\right) + 1$$

$n_T = 33$ bars

$$n_c = n_T \left(\frac{2}{1 + L/B}\right) = 33\left(\frac{2}{1 + 4/3}\right) = 28.28 \approx 29 \text{ bars}$$

$$\text{No. of bars in outer band} = \frac{n_T - n_c}{2} = \frac{33 - 29}{2} = 2 \text{ bars}$$


16ϕ@175 c/c
2 bars
29 bars
2 bars

17. Working Stress Method

17.1 Introduction:

WSM design of RCC member is the oldest method of design. Main advantages of WSM are following

i) Less deflection due to larger section size

ii) Less crack width due to lower stress level of steel.

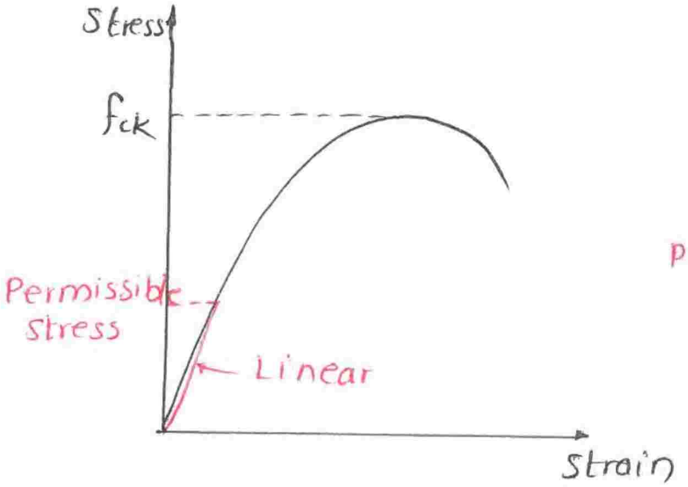
iii) Low % of steel because of larger section size.

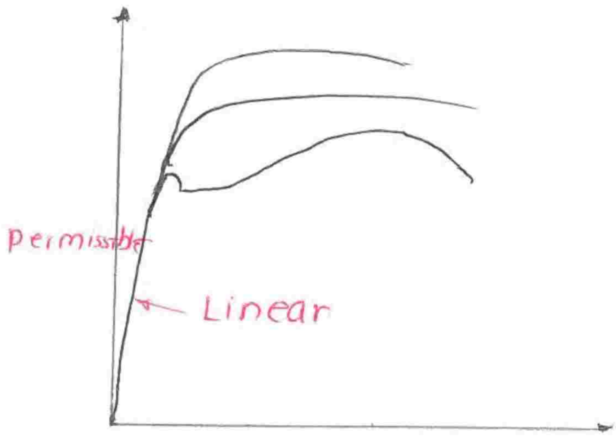
*Note:

LSM is economical than WSM.

17.2 Assumptions:

- Plane section remains plane after bending.
- Tensile strength of concrete is ignored.
- Modular ratio is $\frac{280}{3\sigma_{cbc}}$.
- Both materials are assumed to be linearly elastic.





17.3 Permissible Stress of Material:

A) Concrete:

Grade	σ_{cc}	σ_{cbc}	σ_t
M10	2.5	3.0	1.2
M15	4.0	5.0	2.0
M20	5.0	7.0	2.8
M25	6.0	8.5	3.2
M 30	8.0	10.0	3.6
M 35	9.0	11.5	4.0
M 40	10.0	13.0	4.4

σ_{cc} = Permissible stress under direct compression

σ_{cbc} = Permissible stress under bending compression.

σ_t = Under tension.

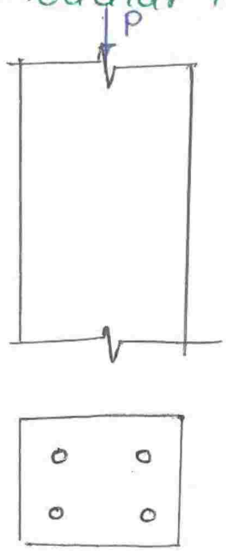
B) Steel (σ_{st})

Types of Stress	Fe 250	Fe 415	Fe 500
• Tension:			
$\phi \leq 20mm$	140	230	275
$\phi > 20mm$	130	230	275
• Compression	130	190	190

* Note:

- σ_{cc} and σ_{cbc} are approximately $\frac{f_{ck}}{4}$ and $\frac{f_{ck}}{3}$ respe.
- Above values are increased by 33.33 % for structure subjected to wind load or earthquake load.

17.4 Use of Modular Ratio:



$$P = P_c + P_s$$

$$\Rightarrow P = f_c \cdot A_c + f_s \cdot A_s \quad \text{......(i)}$$

From strain compatibility.

$$\epsilon_s = \epsilon_c$$

$$\frac{f_s}{E_s} = \frac{f_c}{E_c}$$

$$f_s = \left(\frac{E_s}{E_c}\right) f_c$$

$$f_s = m f_c \quad \text{......(ii)}$$

from (i) and (ii)

$$P = f_c A_c + (m f_c) A_s$$

$$\Rightarrow P = f_c A_c + f_c (m A_s)$$

- From above expression, it is clear that area of steel can be converted into equivalent area of concrete by multiplying modular ratio to area of steel.

Ex. Compare modular ratio of WSM with short term and long term modular ratio. M30 concrete and creep coefficient 1.6

$\Rightarrow$

- WSM

$$m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3 \times 10} = 9.33$$

- Short Term:

$$m = \frac{E_s}{E_c} = \frac{2 \times 10^5}{5000\sqrt{30}} = 7.3$$

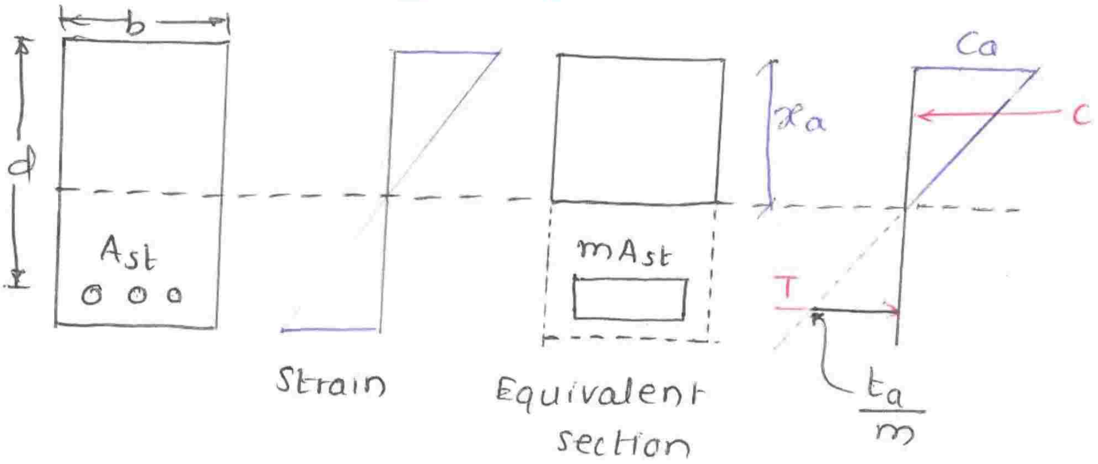
- Long Term

$$m = \frac{E_s}{E_{ce}} = \frac{2 \times 10^5}{\frac{5000\sqrt{30}}{1+1.6}} = 18.98$$

Since, m of WSM lies between short term & long term incorporating the effect of creep.

17.5 Analysis of Singly Reinforced Section:

17.5.1 Position of Neutral Axis



For position of NA

$$C = T$$

$$\Rightarrow \quad \frac{1}{2} \times x_a \times C_a \times b = \frac{t_a}{m} \times m A_{st} \quad \text{----} \quad (i)$$

from stress diagram:

$$\frac{C_a}{x_a} = \frac{t_a/m}{d - x_a}$$

$$\Rightarrow \quad C_a = \left(\frac{x_a}{d - x_a}\right) \cdot \frac{t_a}{m} \quad \text{----} \quad (ii)$$

from (i) and (ii)

$$\Rightarrow \frac{1}{2} \times x_a \times \left(\frac{x_a}{d - x_a}\right) \frac{t_a}{m} \times b = t_a \cdot A_{st}$$

$$\Rightarrow \quad b \cdot x_a \cdot \frac{x_a}{2} = m A_{st} (d - x_a)$$

$\Rightarrow$ Moment of area of comp. zone about NA = Moment of area of tension zone about NA.

Above expression shows that position of NA can be directly calculated by equating moment of area of comp. and tension zone about NA. This cannot be applied in LSM

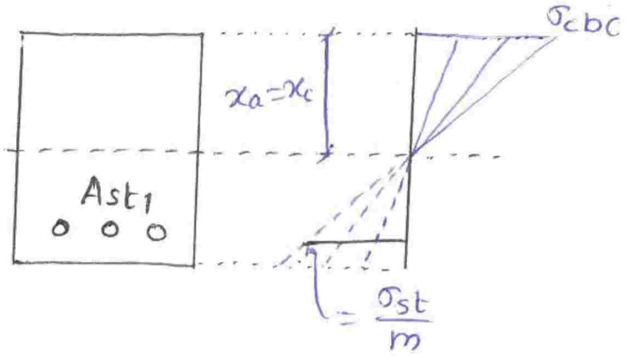
In WSM steel is converted into concrete so above expression is valid.

17.5.2 Types of Section:

Based on quantity of steel present in section, three types of sections are defined.

1) Balanced Section:

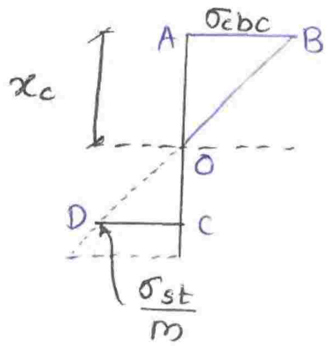
Amount of steel in section is such that concrete and steel both attain their permissible stress simultaneously.



$c_a = \sigma_{cbc}$ $\quad t_a = \sigma_{st}$

$x_a = x_c$

For $x_c \Rightarrow$



$$\frac{OA}{AB} = \frac{OC}{CD}$$

$$\frac{x_c}{\sigma_{cbc}} = \frac{d - x_c}{\sigma_{st}/m}$$

$$\Rightarrow x_c = \left(\frac{m\,\sigma_{cbc}}{m\,\sigma_{cbc} + \sigma_s}\right) d$$

$$\Rightarrow x_c = \left(\frac{mc}{mc + t}\right) \cdot d$$

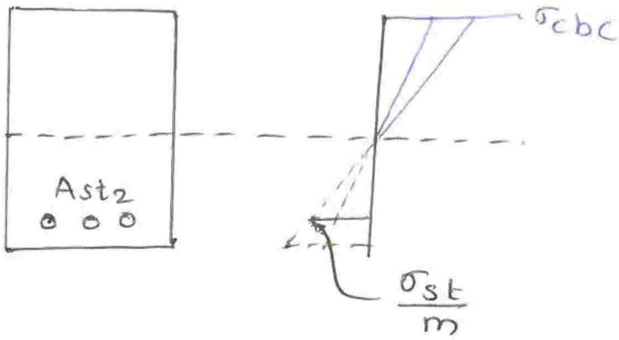
$$\Rightarrow x_c = kd$$

*** Note:**

Position of NA for balanced section (x_c) depends on grade of steel only.

2) Under Reinforced Section:

Amount of steel in section is such that steel attains its permissible stress before concrete.



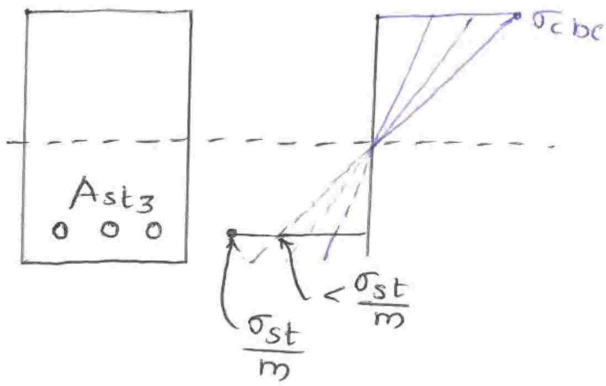
$C_a < \sigma_{cbc}$, $t_a = \sigma_{st}$

$x_a < x_c$

- Failure of under reinforced section is tension failure
- ~~Fail~~Under reinforced section gives sufficient warining before failure so it is preferable.

3) Over Reinforced Section:

Amount of steel in section is such that concrete attains its permissible stress before steel

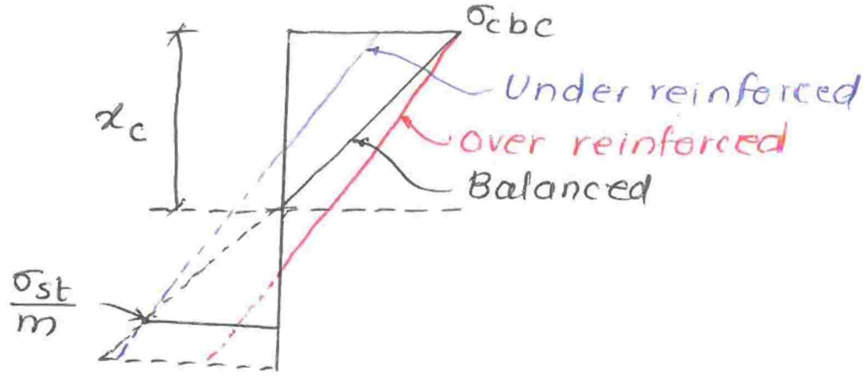


$C_a = \sigma_{cbc}$ $\quad t_a < \sigma_{st}$

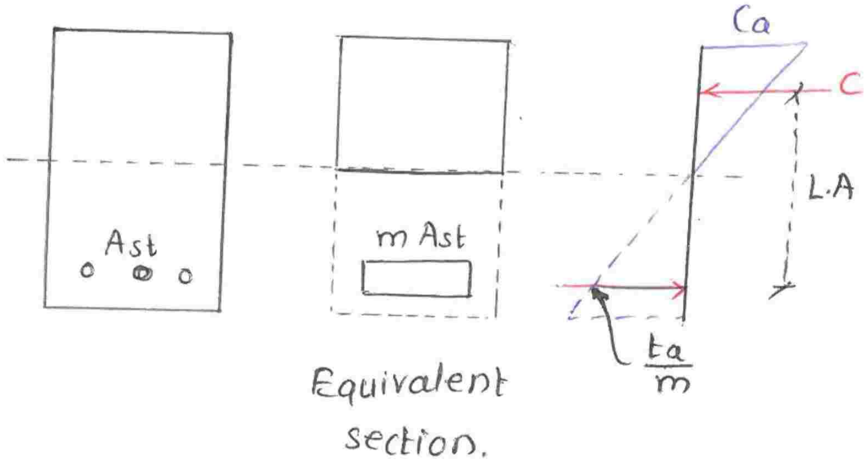
$x_a > x_c$

- Failure of over reinforced section is compression failure
- Failure of over reinforced section is sudden (without warning) so it is undesirable.

* Comparing failure stress diagram of all three types of section.



17.5.3 MR of Section:



$$MR = C \times LA$$

$$= \frac{1}{2} \times x_a \times C_a \times b \left(d - \frac{x_a}{3}\right)$$

$$MR = T \times LA$$

$$= t_a \times A_{st} \times \left(d - \frac{x_a}{3}\right)$$

1) Balanced Section:

$$MR_{bal} = C \times LA$$

$$= \frac{1}{2} \times x_c \times \sigma_{cbc} \times b \left(d - \frac{x_c}{3}\right)$$

$$= \frac{1}{2} \times (kd) \times \sigma_{cbc} \times b \left(d - \frac{kd}{3}\right)$$

$$= \frac{1}{2} \times k \left(1 - \frac{k}{3}\right) \sigma_{cbc}\, bd^2$$

$$= \frac{1}{2} \cdot k \cdot j \cdot \sigma_{cbc} \cdot bd^2$$

$$MR_{bal} = T \times LA$$

$$= \sigma_{st} A_{st} \left(d - \frac{kd}{3}\right)$$

$$= \sigma_{st} \cdot A_{st} \left(1 - \frac{k}{3}\right) d$$

$$MR_{bal} = \sigma_{st} \cdot A_{st} \cdot (jd)$$

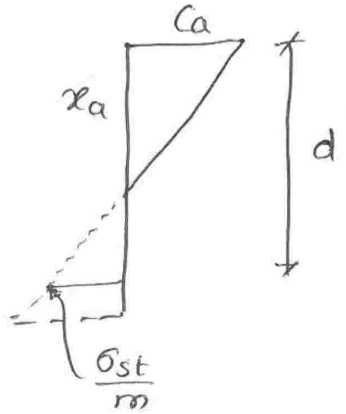
jd = LA of balanced section.

2> Under Reinforced Section:

$$MR = C \times LA$$

$$= \frac{1}{2} \times x_a \times C_a \times b \left(d - \frac{x_a}{3}\right)$$

C_a canbe calculated from stress diagramas follows–



$$C_a = \left(\frac{x_a}{d - x_a}\right) \frac{\sigma_{st}}{m}$$

$$MR = T \times LA$$

$$= \sigma_{st} A_{st} \left(d - \frac{x_a}{3}\right)$$ (Preferable)

3> Over Reinforced Section.

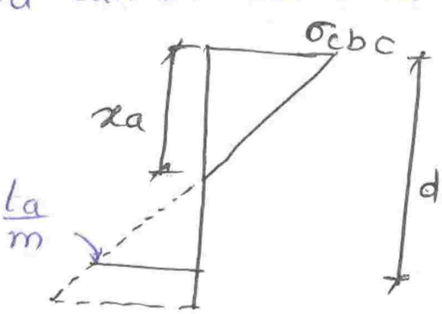
$$MR = C \times LA$$

$$= \frac{1}{2} \times x_a \times \sigma_{cbc} \times b \left(d - \frac{x_a}{3}\right)$$ (Preferable)

$$MR = T \times LA$$

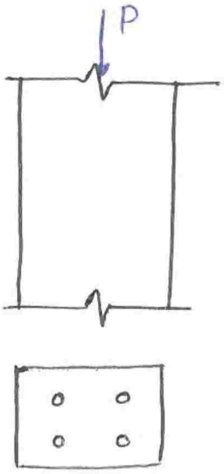
$$= t_a \times A_{st} \cdot \left(d - \frac{x_a}{3}\right)$$

t_a can be calculated from stress diagram as follows:-



$$t_a = \left(\frac{d - x_a}{x_a}\right) m \cdot \sigma_{cbc}$$

17.6 Axial Load Carrying Capacity of Column:



$$P_{short} = P_c + P_s$$

$$P_{short} = \sigma_{cc} \cdot A_c + \sigma_{sc} \cdot A_{sc}$$

$$= \sigma_{cc}(A_g - A_{sc}) + \sigma_{sc} \cdot A_{sc}$$

$$P_{short} = \sigma_{cc} \cdot A_g + (\sigma_{sc} - \sigma_{cc}) \cdot A_{sc}$$

Now,

$$P_{Long} = C_r \cdot P_{short}$$

where, C_r = Reduction coefficient

$$C_r = 1.25 - \frac{L_{eff}}{48b}$$

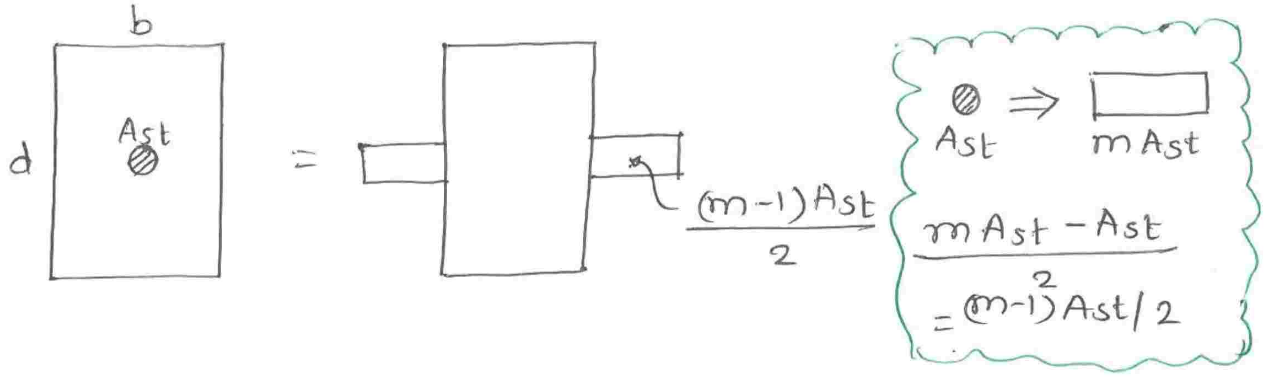
* Note:-

- Modular ratio of compression stee is 1.5 times modular ratio of tension steel This enhancement is done to take care of creep effect of concrete

$$m' = 1.5m$$

$$\text{where, } m = \frac{280}{3\sigma_{cbc}}$$

Ex. Calculate equivalent area of section given below.



$$A_{eq} = bd + (m-1)\cdot A_{st}.$$

Ex. Calculate

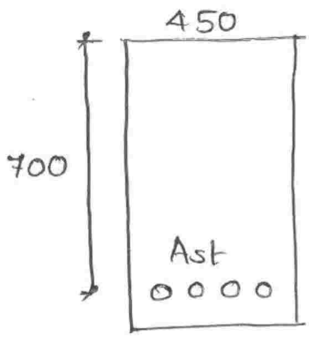
i) Position of critical N.A.

ii) Ast required for balanced section.

iii) MR of balanced section.

iv) If Ast is 3-20φ then calculate MR. also calculate stress of concrete and steel for BM 100 kNm. (WSM special)

v) If Ast is 5-20φ the calculate MR



M20

Fe415.

⇒ (i) $m = \dfrac{280}{3\sigma_{cbc}} = \dfrac{280}{3 \times 7} = 13.33$

$$k = \frac{mc}{mc+t} = \frac{13.33 \times 7}{13.33 \times 7 + 230}$$

$k = 0.288$

$x_c = kd$

$= 0.288 \times 700$

$x_c = 201.6$ mm

(ii)

$$C = T$$

$$\frac{1}{2} \times x_c \times \sigma_{cbc} \times b = \sigma_{st} \cdot A_{st}$$

$$\frac{1}{2} \times 201.6 \times 7 \times 450 = 230 \times A_{st}$$

$$A_{st} = 1380.52 \text{ mm}$$

(iii) $MR_{bal} = \frac{1}{2} k \cdot j \cdot \sigma_{cbc} \, bd^2$

$$= \frac{1}{2} \times 0.288 \times \left(1 - \frac{0.288}{3}\right) \times 7 \times 450 \times 700^2$$

$$MR_{bal} = 200.92 \text{ kN·m}$$

(iv) For position of N.A.

$$b \cdot x_a \cdot \frac{x_a}{2} = m A_{st} \cdot (d - x_a)$$

$$450 \times \frac{x_a^2}{2} = 13.33 \times 3 \times \frac{\pi}{4} \times 20^2 (700 - x_a)$$

$$x_a = 171.74 \text{ mm}$$

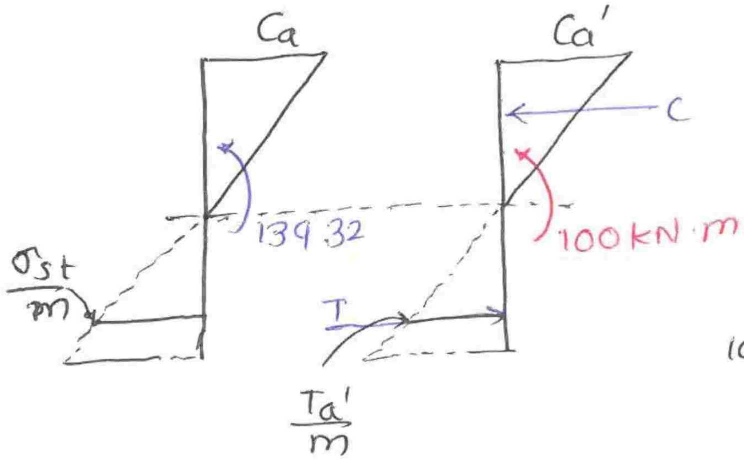
Since, $x_a < x_c$ so section is under reinforced.

$$MR = T \times LA$$

$$= \sigma_{st} \cdot A_{st} \cdot \left(d - \frac{x_a}{3}\right)$$

$$= 230 \times 3 \times \frac{\pi}{4} \times 20^2 \times \left(700 - \frac{171.74}{3}\right)$$

$$MR = 139.32 \text{ kN·m}$$



BM = Resistance of section

$$BM = C \times LA$$

$$BM = \frac{1}{2} \cdot x_a C_a' \cdot b\left(d - \frac{x_a}{3}\right)$$

$$100 \times 10^6 = \frac{1}{2} \times 171.74 \times C_a' \times 450 \times \left(700 - \frac{171.74}{3}\right)$$

From stress diagram:

$$t_a' = \left(\frac{d - x_a}{x_a}\right) m \cdot c_a' = \left(\frac{700 - 171.74}{171.74}\right) \times 13.33 \times 4.03$$

$$t_a' = 165.23 \ N/mm^2$$

v)

For position of N.A.

$$b \cdot x_a \cdot \frac{x_a}{2} = m A_{st} \cdot (d - x_a)$$

$$450 \times \frac{x_a^2}{2} = 13.33 \times 5 \times \frac{\pi}{4} \times 20^2 \times (700 - x_a)$$

$$x_a = 212.906 mm$$

Since, $x_a > x_c$ so section is over reinforced.

$$MR = C \times LA = \frac{1}{2} x_a \cdot \sigma_{cbc} \times b \left(d - \frac{x_a}{3}\right)$$

$$=$$

$$MR = 210.93 \ kN \cdot m$$

STAIRCASE

- **Width – 1 to 2m, Steps – 3 to 12 nos., T+2R = 500, T*R = 40000 to 42000**
- **Residential**
  - **T = 250 to 300**
  - **R = 150 to 180**
- **Public**
  - **T = 250 to 300**
  - **R = 120 to 150**

Width:

1m

2.0m → Public building

Minimum Steps = 3 Nos

Maximum Steps = 12 Nos.

Tread

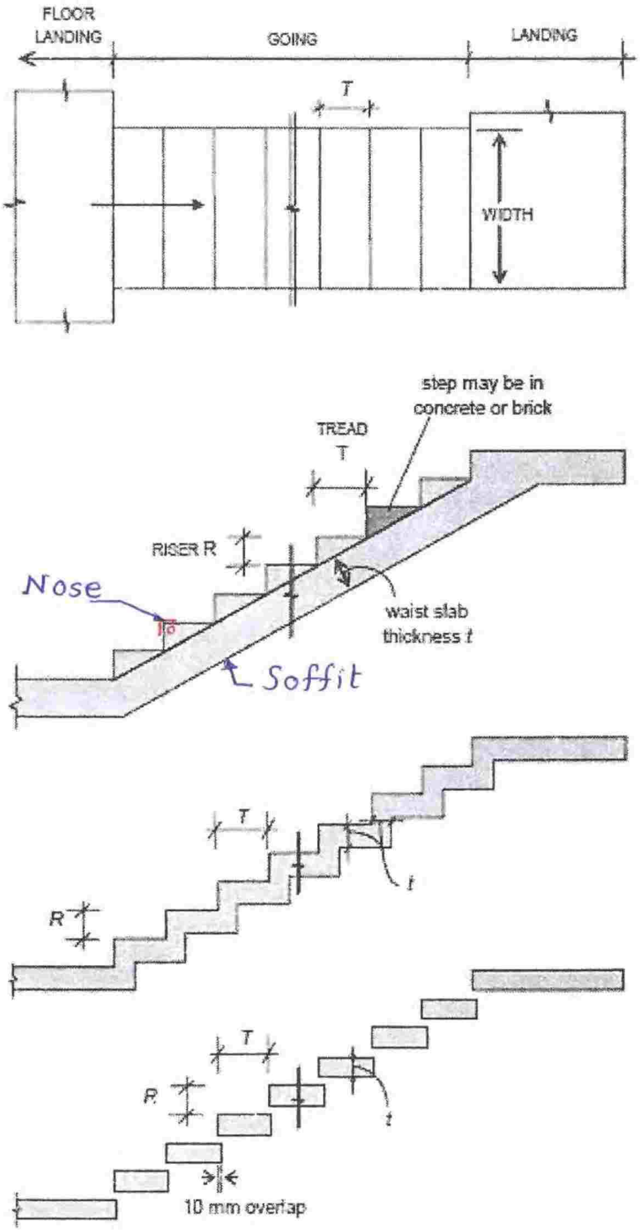
250mm

300mm

Riser:

≥150mm Residential

≤150mm Public



(a) PLAN

(b) 'waist slab' type

(c) 'tread–riser' type

(d) 'isolated tread slab' type

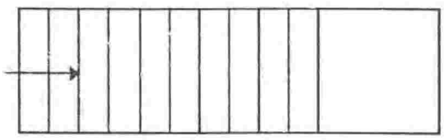
Fig. 12.1 A typical flight in a staircase

12.2 TYPES OF STAIRCASES

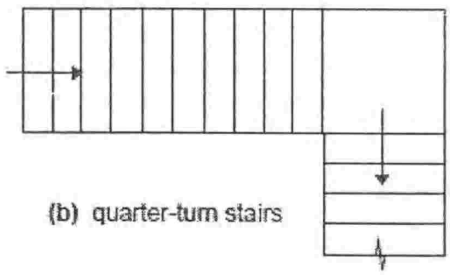
12.2.1 Geometrical Configurations

A wide variety of staircases are met with in practice. Some of the more common geometrical configurations are depicted in Fig. 12.2. These include:

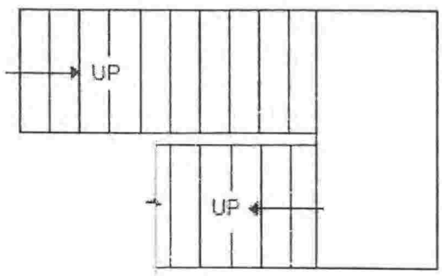
- straight stairs (with or without intermediate landing) [Fig. 12.2(a)]
- quarter-turn stairs [Fig. 12.2(b)]
- dog-legged stairs [Fig. 12.2(c)]
- open well stairs [Fig. 12.2(d)]
- spiral stairs [Fig. 12.2(e)]
- helicoidal stairs [Fig. 12.2(f)]



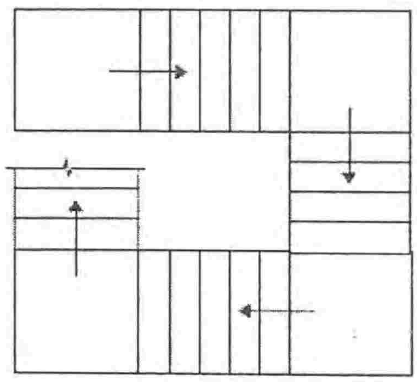
(a) straight stairs



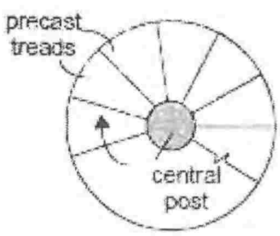
(b) quarter-turn stairs



(c) dog-legged stairs

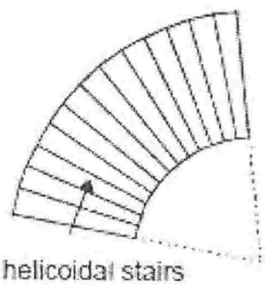


(d) open-well stairs



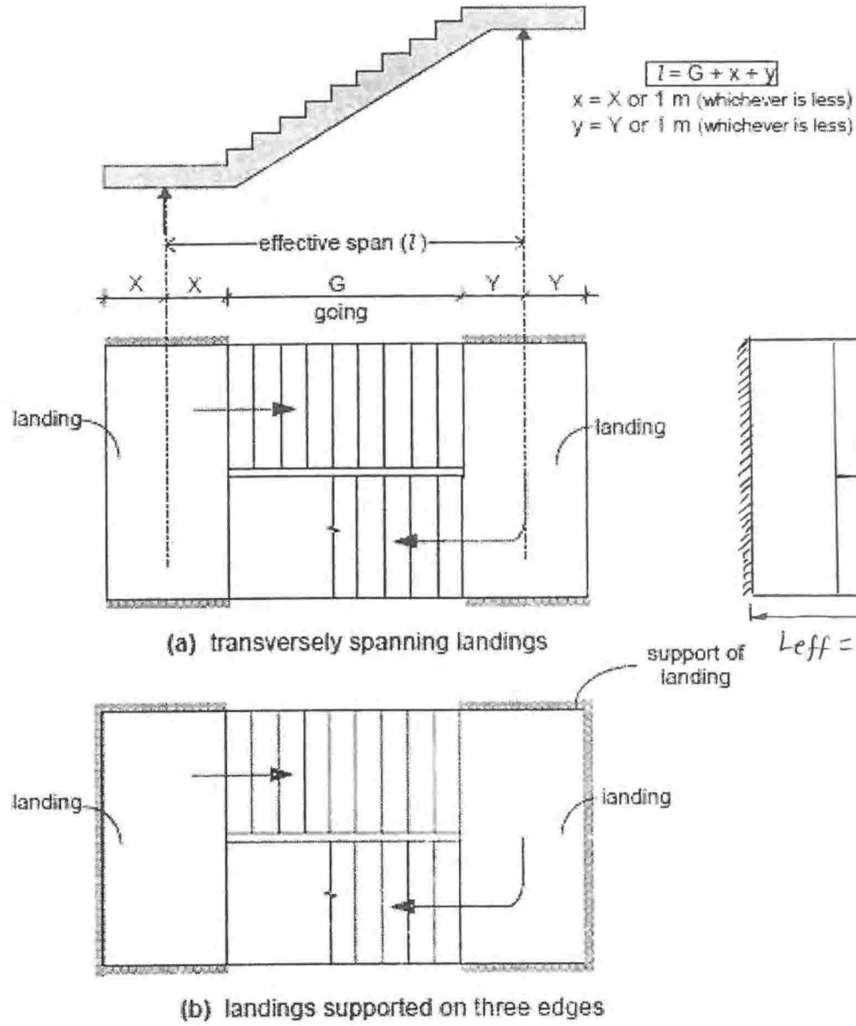
(e) spiral stairs

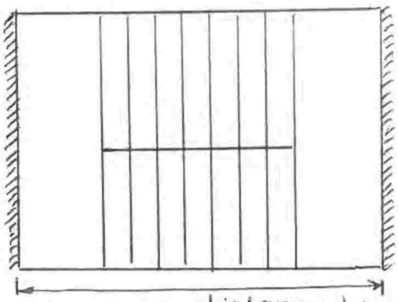
PLAN VIEWS



(f) helicoidal stairs

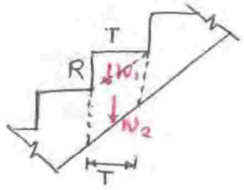
STAIRCASE SPANNING LONGITUDINALLY

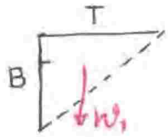




L_{eff} = c/c distance b/w supports

Fig. 12.5 Special support conditions for longitudinally spanning stair slabs





$$W_1 = \frac{\frac{1}{2} \times R \times T \times B \times 25}{T} \text{ kN/m}$$

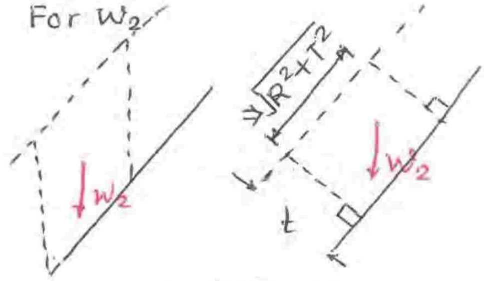
12.3.1 Dead Loads

The components of the dead load to be considered comprise:

- self-weight of stair slab (tread/tread-riser slab/waist slab);
- self-weight of step (in case of 'waist slab' type stairs);
- self-weight of tread finish (usually 0.5 – 1.0 kN/m^2)

The unit weight of reinforced concrete for the slab and step may be taken as 25kN/m^3 as specified in the Code (Cl. 19.2.1).

For W_2



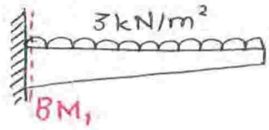
$$W_2 = \frac{\sqrt{R^2+T^2} \times t \times B \times 25}{T} \text{ kN/m}$$

$t =$

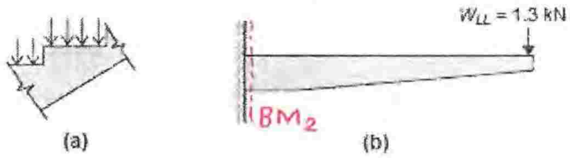
$B =$

12.3.2 Live Loads

Live loads are generally assumed to act as uniformly distributed loads on the horizontal projection of the flight, i.e., on the 'going'. The Loading Code [IS 875 : 1987 (Part II)] recommends a uniformly distributed load of 5.0 kN/m^2 in general, on the going, as well as the landing. However, in buildings (such as residences) where the specified floor live loads do not exceed 2.0 kN/m^2, and the staircases are not liable to be overcrowded, the Loading Code recommends a lower live load of 3.0 kN/m^2 [Fig. 12.6(a)].



$$w_{LL} = \begin{cases} 5.0 \text{ kN/m}^2 & \text{in general} \\ 3.0 \text{ kN/m}^2 & \text{when overcrowding is unlikely} \end{cases}$$



Design BM = Maximum $\begin{cases} BM_1 \\ BM_2 \end{cases}$

Fig. 12.6 Code specifications for live loads on stair slabs

Further, in the case of *structurally independent cantilever steps*, the Loading Code requires the tread slab to be capable of safely resisting a concentrated live load of 1.3 kN applied to the free end of each cantilevered tread [Fig. 12.6(b)].

It may be noted that the specified live loads are *characteristic* loads; these loads as well as the characteristic dead loads should be multiplied by the appropriate *load factors* in order to provide the *factored loads* required for 'limit state design'.

In the case of stairs with open wells, where spans partly crossing at right angles occur, the load on areas common to any two such spans may be taken as one-half in each direction as shown in Fig. 18. Where flights

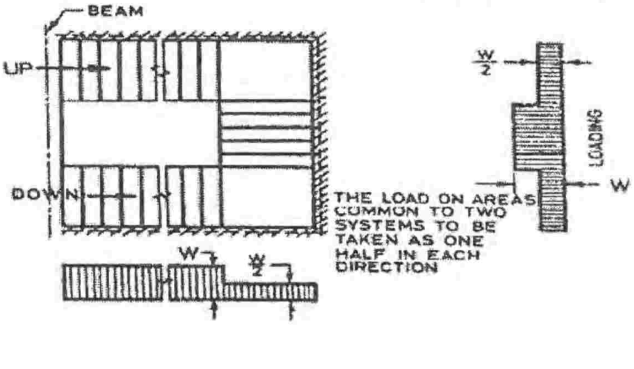
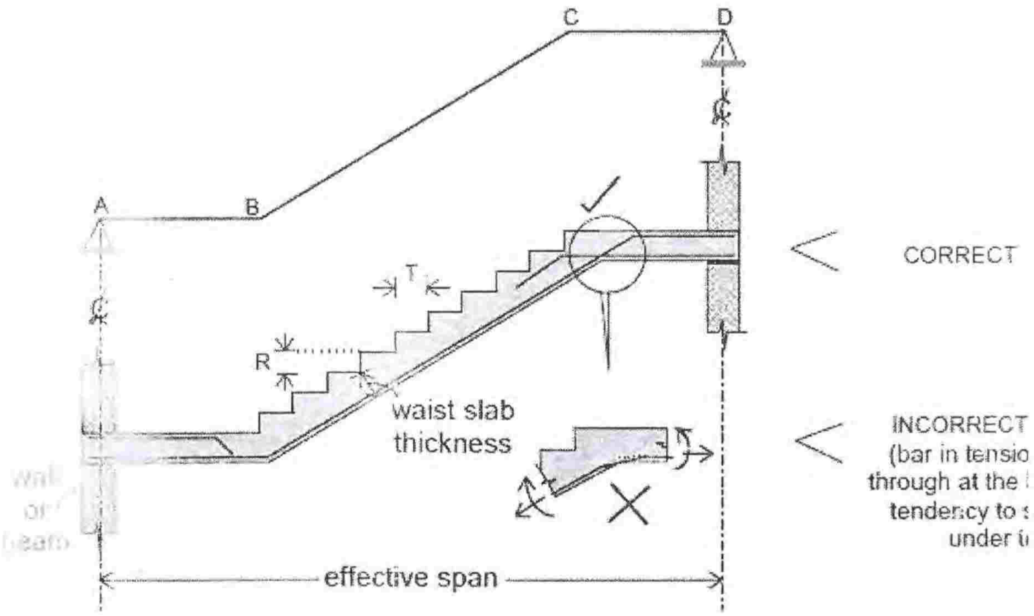


FIG. 18 LOADING ON STAIRS WITH OPEN WELLS



EXAMPLE 12.5

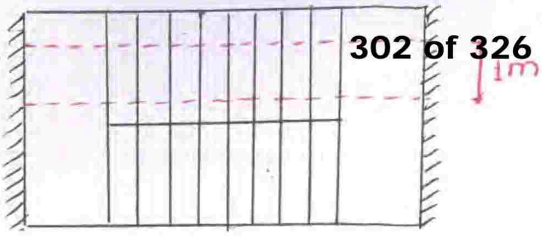
Design a ('waist slab' type) dog-legged staircase for an office building, given the following data:

- height between floor = 3.2 m;
- riser = 160 mm, tread = 270 mm;
- width of flight = landing width = 1.25 m
- live load = 5.0 kN/m^2
- finishes load = 0.6 kN/m^2

Assume the stairs to be supported on 230 mm thick masonry walls at the outer edges of the landing, parallel to the risers [Fig. 12.13(a)]. Use M 20 concrete and Fe 415 steel. Assume *mild* exposure conditions.

SOLUTION

- Given: $R = 160$ mm, $T = 270$ mm $\Rightarrow \sqrt{R^2+T^2} = 314$ mm
  Effective span = c/c distance between supports = 5.16 m [Fig. 12.13(a)].
- Assume a waist slab thickness $\approx l/20 = 5160/20 = 258 \rightarrow 260$ mm.
  based on experience
  Assuming 20 mm clear cover (*mild* exposure) and 12 ϕ main bars, effective depth $d = 260 - 20 - 12/2 = 234$ mm.
  The slab thickness in the landing regions may be taken as 200 mm as the bending moments are relatively low here.
  D



A strip of unit width is designed

- *Loads on going* [Ref. 12.13(b)] on projected plan area:
  - (1) self-weight of waist slab @ $25 \times 0.26 \times 314/270$ $= 7.56$ kN/m^2
  - (2) self-weight of steps @ $25 \times \left(\frac{1}{2} \times 0.16\right)$ $= 2.00$ "
  - (3) finishes (given) $= 0.60$ "
  - (4) live load (given) $= 5.00$ "

  15.16 kN/m^2

  $\Rightarrow$ Factored load $= 15.16 \times 1.5 = 22.74$ kN/m^2
- *Loads on landing*
  - (1) self-weight of slab @ $25 \times 0.20 = 5.00$ kN/m^2
  - (2) finishes @ 0.6 "
  - (3) live loads @ 5.0 "

  10.60 kN/m^2

  $\Rightarrow$ Factored load $= 10.60 \times 1.5 = 15.90$ kN/m^2
- *Design Moment* [refer Fig. 12.13(b)]

  Reaction $R = (15.90 \times 1.365) + (22.74 \times 2.43)/2 = 49.33$ kN/m

  Maximum moment at midspan:

  $$M_u = (49.33 \times 2.58) - (15.90 \times 1.365) \times (2.58 - 1.365/2) - (22.74) \times (2.58 - 1.365)^2/2$$

  $$= 69.30 \text{ kNm/m}$$
- *Main reinforcement*

  $$R = \frac{M_u}{bd^2} = \frac{69.30 \times 10^6}{10^3 \times 234^2} = 1.265 \text{ MPa}$$

  $M_{u,lim} = 0.138 f_{ck} b d^2$

  $= 0.138 \times 20 \times 1000 \times 234^2$

  Assuming $f_{ck} = 20$ MPa, $f_y = 415$ MPa,

  $M_{u,lim} = 151.12$ kNm $\Rightarrow$ So, $M_u < M_{u,lim}$

  $$\frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{20}{2 \times 415}\left[1 - \sqrt{1 - 4.598 \times 1.265/20}\right] = 0.381 \times 10^{-2}$$

  [This may also be obtained from design aids Table 3(a)].

  $$\Rightarrow (A_{st})_{reqd} = (0.381 \times 10^{-2}) \times 10^3 \times 234 = 892 \text{ mm}^2/\text{m}$$

  $$A_{st} = \frac{0.5 f_{ck} b d^2}{f_y}\left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} b d^2}}\right]$$

  $$= \frac{0.5 \times 20 \times 1000 \times 234}{415} \times \left[1 - \sqrt{1 - \frac{4.6 \times 69.3 \times 10^6}{20 \times 1000 \times 234^2}}\right]$$

  $A_{st} = 892$ mm^2

  Required spacing of 12 ϕ bars $= \frac{113 \times 10^3}{892} = 127$ mm

  Required spacing of 16 ϕ bars $= \frac{201 \times 10^3}{892} = 225$ mm (to be reduced slightly to account for reduced effective depth)

  Provide 16 ϕ @ 220c/c
- *Distributors*

  $(A_{st})_{reqd} = 0.0012\, bt$ (for Fe 415 bars)

  $= 0.0012 \times 10^3 \times 260 = 312$ mm^2/m

  spacing 10 ϕ bars $= 78.5 \times 10^3 / 312 = 251$ mm

  Provide 10 ϕ @ 250c/c as distributors